'समानो मन्त्रः समितिः समानी'

# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2023

## GE1-P1-MATHEMATICS

## (REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

*The figures in the margin indicate full marks.*

**The question paper contains GE1 and GE4. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.**

### GE1

### CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

### GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Evaluate: $\lim\limits_{x\to 0}\dfrac{x^2+\sin x^2}{x^2+x^3}$ 3

(b) Find the asymptotes of the hyperbola $\dfrac{(y+4)^2}{3}-\dfrac{(x-2)^2}{5}=1$. 3

(c) Show that the curve $y=e^{-x^2}$ has points of inflexion at $x=\pm\dfrac{1}{\sqrt{2}}$. 3

(d) Find the arc length of the curve $x=e^{2t}\sin 2t$, $y=e^{2t}\cos 2t$ from $t=0$ to $t=\pi$. 3

(e) If $ax+by$ transforms to $a'x'+b'y'$ due to rotation of axes, then show that $a^2+b^2=a'^2+b'^2$. 3

(f) Examine whether the equation $(a^2-2xy-y^2)\,dx-(x+y)^2\,dy=0$, is exact. If it be exact, then find the primitive. 3

### GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) If $y=\cos(m\sin^{-1}x)$ prove that $(1-x^2)y_{n+2}-(2x+1)xy_{n+1}+(m^2-n^2)y_n=0$ and hence find $y_n(0)$. 6

(b) Reduce the equation $6x^2-5xy-6y^2+14x+5y+4=0$ to their canonical form and determine the nature of the conics represented by it. 6

(c) Find the equation of the sphere having the circle $x^2+y^2+z^2=9$, $x+y-2z=4$ as a great circle. 6

(d) Evaluate $\int \operatorname{cosec}^5 x\, dx$ by using reduction formula. 6

(e) Solve the equation $y^2(y-xp)=x^4p^2$ and find its singular solution. 6

(f) Solve: $\dfrac{dy}{dx}-\dfrac{\tan y}{1+x}=(1+x)e^x\cdot\sec y$ 6

## GROUP-C

**Answer any *two* questions** 12×2 = 24

3. (a) Find the envelops of the family of ellipses $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, where the parameters $a$ and $b$ are connected by $a^3+b^3=c^3$, $c$ being a constant. 6

(b) Show that the straight line $l/r=A\cos\theta+B\sin\theta$ touches the conic $l/r=1+e\cos(\theta-\gamma)$ if $A^2+B^2-2e(A\cos\gamma+B\sin\gamma)+e^2=1$. 6

4. (a) If $I_n=\int_0^a(a^2-n^2)^n\,dx$, $n>0$. Prove that $(2n+1)I_n=2na^2I_{n-1}$. 6

(b) Find the area of the region bounded by one arch of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ and the $x$ axis. 6

5. (a) Find the equation of the cylinder whose generators are parallel to the line $\dfrac{x}{1}=\dfrac{y}{-2}=\dfrac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1,\ z=3$. 6

(b) Find the volume of the solid generated by revolving the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ about its major axis. 6

6. (a) Find the integrating factor of $(3x^2y^4+2xy)\,dx+(2x^3y^3-x^2)\,dy=0$ and hence solve. 2+4

(b) Solve: $\dfrac{dy}{dx}=\dfrac{y-x+1}{y+x+5}$ 6

## GE4

## GROUP THEORY

## GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Prove that every group of prime order is cyclic. 3

(b) Find the number of elements of order 5 in the group $(\mathbb{Z}_{20}, +)$. 3

(c) Prove that a group $(G,\circ)$ is abelian if and only if $(a\circ b)^{-1}=a^{-1}\circ b^{-1}$ for all $a,b\in G$. 3

(d) Show that the groups $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$ are not isomorphic. 3

(e) Let $G$ be a commutative group. If $a$ and $b$ are two distinct elements of $G$ such that $o(a)=2=o(b)$, show that $|\langle a,b\rangle|=4$. 3

(f) Show that center of group is a normal subgroup of that group. 3

## GROUP-B

2. Answer any ***four*** questions: $6\times4=24$

(a) Let $G$ be a finite cyclic group of order $m$. Then for every positive divisor $d$ of $m$, prove that there exists a unique subgroup of $G$ of order $d$. 6

(b) Let $H$ and $K$ be subgroup of a group $G$. Prove that $HK$ is a subgroup of $G$ if and only if $HK = KH$. 6

(c) Show that every finite cyclic group of order $n$ is isomorphic to $(\mathbb{Z}_n, +)$ and every infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$. 6

(d) Find all homomorphism from $(\mathbb{Z}_8, +)$ to $(\mathbb{Z}_6, +)$. 6

(e) State and prove Lagrange's Theorem. 6

(f) Let $G=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R},\ ad-bc=1\right\}$. Show that $G$ is a group under usual matrix multiplication. 6

## GROUP-C

3. Answer any ***two*** questions: $12\times2=24$

(a) (i) Prove that every subgroup of a cyclic group is cyclic. 4

(ii) Let $H$ be a subgroup of $G$ and $[G:H]=2$. Prove that 4+2

(I) $\forall\, x \in G,\ x^2 \in H$

(II) $H$ is normal subgroup of $G$.

(iii) Give an example of a non-commutative group in which every subgroup is normal. 2

(b) (i) Let $H$ be a cyclic subgroup of $G$. If $H$ is normal in $G$, then prove that every subgroup of $H$ is normal in $G$. 6

(ii) Let $\phi : (G, \circ) \to (G', *)$ be a homomorphism. Then show that $\phi$ is one-one iff $\ker\phi = \{e_G\}$. 4

(iii) Show that $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{R}, +)$. 2

(c) (i) Prove that if $H$ is a subgroup of a cyclic group $G$, then the quotient group $G/H$ is cyclic. 3

(ii) Let $G$ be a group and $a \in G$ such that $o(a)=n$. Then prove that $a^m = e$ if and only if $m = nq$ $(m, q \in \mathbb{Z})$. 4

(iii) State and prove the fundamental theorem of group homomorphism. 5

(d) (i) Find all homomorphism from the group $(\mathbb{Z}_6, +)$ to $(\mathbb{Z}_4, +)$. 4

(ii) Let $G$ be a finite group generated by $a$. Prove that $o(G)=n$ iff $o(a)=n$. 4

(iii) Let $G$ be an infinite cyclic group generated by $a$. Prove that $a$ and $a^{-1}$ are only generators of the group. 4

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# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2023

## GE1-P1-MATHEMATICS

## (OLD SYLLABUS 2018)

Time Allotted: 2 Hours — Full Marks: 60

*The figures in the margin indicate full marks.*

**The question paper contains GE1, GE2, GE3, GE4 and GE5. Candidates are required to answer any *one* from the *five* courses and they should mention it clearly on the Answer Book.**

### GE1

### CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

### GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) Evaluate: $\lim\limits_{x\to0}\dfrac{x^2+\sin x^2}{x^2+x^3}$ — 3

(b) Find the asymptotes of the hyperbola $\dfrac{(y+4)^2}{3}-\dfrac{(x-2)^2}{5}=1$. — 3

(c) Show that the curve $y=e^{-x^2}$ has points of inflexion at $x=\pm\dfrac{1}{\sqrt{2}}$. — 3

(d) Find the arc length of the curve $x=e^{2t}\sin 2t$, $y=e^{2t}\cos 2t$ from $t=0$ to $t=\pi$. — 3

(e) If $ax+by$ transforms to $a'x'+b'y'$ due to rotation of axes, then show that $a^2+b^2=a'^2+b'^2$. — 3

(f) Examine whether the equation $(a^2-2xy-y^2)\,dx-(x+y)^2\,dy=0$, is exact. If it be exact, then find the primitive. — 3

### GROUP-B

2. Answer any ***four*** questions: $6\times4 = 24$

(a) If $y=\cos(m\sin^{-1}x)$ prove that $(1-x^2)y_{n+2}-(2x+1)xy_{n+1}+(m^2-n^2)y_n=0$ and hence find $y_n(0)$. — 6

(b) Reduce the equation $6x^2-5xy-6y^2+14x+5y+4=0$ to their canonical form and determine the nature of the conics represented by it. — 6

(c) Find the equation of the sphere having the circle $x^2+y^2+z^2=9$, $x+y-2z=4$ as a great circle. — 6

(d) Evaluate $\int \text{cosec}^5 x\, dx$ by using reduction formula. 6

(e) Solve the equation $y^2(y - xp) = x^4 p^2$ and find its singular solution. 6

(f) Solve: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \cdot \sec y$ 6

## GROUP-C

**Answer any *two* questions** 12×2 = 24

3. (a) Find the envelops of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where the parameters $a$ and $b$ are connected by $a^3 + b^3 = c^3$, $c$ being a constant. 6

(b) Show that the straight line $l/r = A\cos\theta + B\sin\theta$ touches the conic $l/r = 1 + e\cos(\theta - \gamma)$ if $A^2 + B^2 - 2e(A\cos\gamma + B\sin\gamma) + e^2 = 1$. 6

4. (a) If $I_n = \int_0^a (a^2 - n^2)^n\, dx$, $n > 0$. Prove that $(2n+1)I_n = 2na^2 I_{n-1}$. 6

(b) Find the area of the region bounded by one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and the $x$ axis. 6

5. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. 6

(b) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its major axis. 6

6. (a) Find the integrating factor of $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$ and hence solve. 2+4

(b) Solve: $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$ 6

## GE2

### ALGEBRA

## GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Find the value of $(1+i)^{1/5}$.

(b) Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4 + 16x^2 + 7x - 11 = 0$.

(c) If $x + \frac{1}{x} = 2\cos\frac{\pi}{7}$, then find the value of $x^7 + \frac{1}{x^7}$.

(d) Find $A^{100}$, where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

(e) If $1, 1, \alpha$ are roots of $x^3 - 6x^2 + 9x - 4 = 0$, then find the value of $\alpha$.

(f) Prove that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

## GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) Solve the following equation:

$$x^3 - 18x - 35 = 0, \quad \text{by Cardan's method}$$

(b) If $n$ be a positive integer, then prove that

$$\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right),$$

where $\theta$ is real.

(c) Show that the mapping $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$, defined by $f(x) = (x+1)/(x-1)$ is bijective. Determine $f^{-1}$. 3+3

(d) Find the condition that the roots of the equation $x^3 - px^2 + qx - \gamma = 0$ will be in A.P.

(e) Prove that $8 \mid 7^n + 3^n - 2$ for all positive integers $n$.

(f) Determine the value of $a$ and $b$ for which the system

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + az = b^2$$

has no solution.

## GROUP-C

**Answer any *two* questions** 12×2 = 24

3. (a) Show that 6

$$\left(\frac{s-a_1}{n-1}\right)^{a_1}\left(\frac{s-a_2}{n-1}\right)^{a_2}\cdots\cdots\left(\frac{s-a_n}{n-1}\right)^{a_n} \le \left(\frac{s}{n}\right)^s$$

where $a_1, a_2, \cdots, a_n$ are positive rational numbers and $s = a_1 + a_2 + \cdots\cdots + a_n$.

(b) Prove that $1^n - 3^n - 6^n + 8^n$ is divisible by 10, $\forall n \in \mathbb{N}$. 6

4. (a) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are $\left(\frac{1}{\beta} + \frac{1}{\gamma} - \frac{1}{\alpha}\right), \left(\frac{1}{\gamma} + \frac{1}{\alpha} - \frac{1}{\beta}\right), \left(\frac{1}{\alpha} + \frac{1}{\beta} - \frac{1}{\gamma}\right)$. 6

(b) Let the equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots. Show that $(bc - ad)^2 = 4(b^2 - ac)(c^2 - bd)$ and the equal root is $\dfrac{1}{2}\dfrac{bc - ad}{ac - b^2}$. 6

5. (a) Find the eigen values and corresponding eigen vectors of the following matrix: 6

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$$

(b) Find all real $x$ for which the rank of matrix $\begin{pmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{pmatrix}$ is less than 4. 6

6. (a) Determine the values of $\lambda$ and $\mu$, when the following equations: 6

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has (i) no solution (ii) an infinite numbers of solutions (iii) unique solution.

(b) Reduce the matrix $A$ to its normal form and hence find its rank, where 6

$$A=\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{pmatrix}$$

## GE3

## DIFFERENTIAL EQUATION AND VECTOR CALCULUS

## GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Show that the function $f(x,\ y)=y^{2/3}$ does not satisfy the Lipschitz condition on the rectangle $|x|\le 1,\ |y|\le 1$.

(b) Show that $\sin x,\ \sin 2x,\ \sin 3x$ are linearly independent on $I=[0,\ 2\pi]$.

(c) Find a particular solution of $4x^2\dfrac{d^2y}{dx^2}+8x\left(\dfrac{dy}{dx}\right)+y=4x$.

(d) Show that $x=0$ is an ordinary point of $(x^2-1)y''+xy'-y=0$, but $x=1$ is a regular singular point.

(e) If $\vec{r}=a\cos t\hat{i}+a\sin t\hat{j}+bt\hat{k}$, show that $\left|\dfrac{d\vec{r}}{dt}\times\dfrac{d^2\vec{r}}{dt^2}\right|^2=a^2(a^2+b^2)$.

(f) Examine whether the vector valued function $\vec{r}=t^2\hat{i}+e^t\hat{j}+\dfrac{1}{t+3}\hat{k}$ is continuous at $t=-3$ or not.

## GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) State and prove superposition principle for homogeneous equation.

(b) Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2}+4y=xe^x$.

(c) Solve: $\dfrac{dx}{dt}+4x+3y=t$

$\dfrac{dy}{dt}+2x+5y=e^t$

(d) Find the series solution of the differential equation $2x^2y''-xy'+(1-x^2)y=0$ near $x=0$.

(e) Prove that for any four vectors $\vec{a},\vec{b},\vec{c},\vec{d}$;

$$(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{d})+(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=0$$

(f) Find the volume of the tetrahedron whose vertices are $(2,1,3)$, $(4,-1,-3)$, $(-5,4,1)$ and $(-1,3,5)$.

## GROUP-C

**Answer any *two* questions** 12×2 = 24

3. (a) If $\vec{r}_1(t)=t\hat{i}-3t\hat{j}+2t^2\hat{k}$, $\vec{r}_2(t)=\hat{i}-2t\hat{j}+2\hat{k}$, $\vec{r}_3(t)=3t^2\hat{i}+(2-6t)\hat{j}-4t\hat{k}$ then find $\int_1^2\vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2)\,dt$. 6

(b) Solve by the method of undetermined coefficients: 6

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=8e^{2x}$$

4. (a) Examine whether the differential equation $(y^2e^x+2xy)\,dx-x^2dy=0$ is exact. If exact, then solve it. 6

(b) Show that the Wronskian of the two solutions of the equation 6

$$P_0(x)y''+P_1(x)y'+P_2(x)y=0$$

is either identically zero or never zero $\forall\, x\in[a,b]$ where $P_0(x)\neq 0$ and $P_0(x)$, $P_1(x)$, $P_2(x)$ are continuous functions on the given interval.

5. (a) Solve: $x^2\dfrac{d^2y}{dx^2}+5x\dfrac{dy}{dx}+3y=e^{-x}$ 6

(b) Show that the four points with position vectors $\vec{\alpha},\vec{\beta},\vec{\gamma},\vec{\delta}$ are coplanar if and only if $[\vec{\beta}\,\vec{\gamma}\,\vec{\delta}]+[\vec{\gamma}\,\vec{\alpha}\,\vec{\delta}]+[\vec{\alpha}\,\vec{\beta}\,\vec{\delta}]=[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]$. 6

6. (a) Prove that the necessary and sufficient condition for a vector $\vec{r} = \overrightarrow{f(t)}$ to have a constant magnitude if $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$. 6

(b) If $\vec{A} = (3x^2 + 6y)\hat{i} + 14yz\hat{j} + 20x^2z^2\hat{k}$, then evaluate $\int_C \vec{A} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the path $x = t$, $y = t^2$, $z = t^3$. 6

## GE4

## GROUP THEORY

### GROUP-A

1. Answer any ***four*** questions: $3 \times 4 = 12$

(a) Prove that every group of prime order is cyclic. 3

(b) Find the number of elements of order 5 in the group $(\mathbb{Z}_{20}, +)$. 3

(c) Prove that a group $(G, \circ)$ is abelian if and only if $(a \circ b)^{-1} = a^{-1} \circ b^{-1}$ for all $a, b \in G$. 3

(d) Show that the groups $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$ are not isomorphic. 3

(e) Let $G$ be a commutative group. If $a$ and $b$ are two distinct elements of $G$ such that $o(a) = 2 = o(b)$, show that $|\langle a, b\rangle| = 4$. 3

(f) Show that center of group is a normal subgroup of that group. 3

### GROUP-B

2. Answer any ***four*** questions: $6 \times 4 = 24$

(a) Let $G$ be a finite cyclic group of order $m$. Then for every positive divisor $d$ of $m$, prove that there exists a unique subgroup of $G$ of order $d$. 6

(b) Let $H$ and $K$ be subgroup of a group $G$. Prove that $HK$ is a subgroup of $G$ if and only if $HK = KH$. 6

(c) Show that every finite cyclic group of order $n$ is isomorphic to $(\mathbb{Z}_n, +)$ and every infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$. 6

(d) Find all homomorphism from $(\mathbb{Z}_8, +)$ to $(\mathbb{Z}_6, +)$. 6

(e) State and prove Lagrange's Theorem. 6

(f) Let $G = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} :\ a, b, c, d \in \mathbb{R},\ ad - bc = 1\right\}$. Show that $G$ is a group under usual matrix multiplication. 6

### GROUP-C

3. Answer any ***two*** questions: $12 \times 2 = 24$

(a) (i) Prove that every subgroup of a cyclic group is cyclic. 4

(ii) Let $H$ be a subgroup of $G$ and $[G : H] = 2$. Prove that 4+2

(I) $\forall\, x \in G,\ x^2 \in H$

(II) $H$ is normal subgroup of $G$.

(iii) Give an example of a non-commutative group in which every subgroup is normal. 2

(b) (i) Let $H$ be a cyclic subgroup of $G$. If $H$ is normal in $G$, then prove that every subgroup of $H$ is normal in $G$. 6

(ii) Let $\phi:(G,\circ)\to(G',*)$ be a homomorphism. Then show that $\phi$ is one-one iff $\ker\phi=\{e_G\}$. 4

(iii) Show that $(\mathbb{Q},+)$ is not isomorphic to $(\mathbb{R},+)$. 2

(c) (i) Prove that if $H$ is a subgroup of a cyclic group $G$, then the quotient group $G/H$ is cyclic. 3

(ii) Let $G$ be a group and $a\in G$ such that $o(a)=n$. Then prove that $a^m=e$ if and only if $m=nq$ $(m,q\in\mathbb{Z})$. 4

(iii) State and prove the fundamental theorem of group homomorphism. 5

(d) (i) Find all homomorphism from the group $(\mathbb{Z}_6,+)$ to $(\mathbb{Z}_4,+)$. 4

(ii) Let $G$ be a finite group generated by $a$. Prove that $o(G)=n$ iff $o(a)=n$. 4

(iii) Let $G$ be an infinite cyclic group generated by $a$. Prove that $a$ and $a^{-1}$ are only generators of the group. 4

## GE5

### NUMERICAL METHODS

### GROUP-A

1. Answer any ***four*** questions: $3\times4=12$

(a) Find the number of significant figures in $V_A=2.3951$, given its relative error as $0.1\times10^{-3}$.

(b) Give the geometrical interpretation of Regula-Falsi method.

(c) Prove that $\Delta\nabla=\Delta-\nabla$, where the symbols have their usual meaning.

(d) Compute by the method of iteration the positive roots of the equation $x^3+x-5=0$ correct up to 3 significant figures.

(e) Find the interpolating polynomial which interpolates $f(x)$ such that $f(0)=-1$, $f(1)=0$, $f(2)=5$.

(f) Show that $\Delta\log f(x)=\log\left\{1+\frac{\Delta f(x)}{f(x)}\right\}$.

### GROUP-B

2. Answer any ***four*** questions: $6\times4=24$

(a) Deduce Newton's forward interpolation formula for equally spaced arguments.

(b) Using divided difference formula, find a real root of the equation $x^2-\sin x=0$.

(c) Evaluate $\int_{0.1}^{0.7}(e^x+2x)\,dx$, by Simpson's $\frac{1}{3}$rd rule with step length $h=0.1$, correct upto two decimal places.

(d) Discuss the Bisection method for finding a simple real root of $f(x)=0$.

(e) Compute $y(0.2)$ from the equation $\frac{dy}{dx}=x^2-y$ ; $y(0)=1$, with $h=0.1$ by Runge-Kutta method correct up to 2 decimal places.

(f) Explain Gauss-Jacobi method for solving a system of linear equations. State sufficient conditions for the convergence of this method.

## GROUP-C

**Answer any *two* questions** 12×2 = 24

3\. (a) Solve the following system of equations by Gauss-Seidel method 6

$$8x+2y-z=8$$
$$2x-8y+3z=6$$
$$x-y-z=-4$$

correct up to two decimal places.

(b) Show that Newton-Raphson method is quadratically convergent. 6

4\. (a) Deduce the modified Euler's method for solving the following differential equation: 6

$$\frac{dy}{dx}=f(x,\ y)\ ,\qquad y(x_0)=y_0$$

(b) Explain the method of fixed point iteration with the condition of convergence for numerical solution of an equation of the form $x=\phi(x)$. 6

5\. (a) Solve by using suitable interpolation formula to find $f(10)$ correct upto 3 decimal places from: 6

| $x$ | 5 | 6 | 9 | 11 |
|---|---|---|---|---|
| $f(x)$ | 12 | 13 | 14 | 16 |

(b) Show that $\Delta^4 y_0 = y_4-4y_3+6y_2-4y_1+y_0$, where $y_r = y(x_0+rh)$, where $h$ is the length of equal subintervals. Further show that the 3rd order difference of $f(x)=2x^2-3x+1$ is zero. 4+2

6\. (a) Prove that $\Delta^n\left(\frac{1}{x}\right)=\frac{(-1)^n\, n!\, h^n}{x(x+h)\cdots(x+nh)}$ for any positive integer $n$, where $\Delta$ being forward difference operator and $h$ is the step length. 6

(b) Establish, Newton-Cotes' numerical integration formula. 6

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