'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2023

CC1-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: 1×5 = 5

   (a) Evaluate: $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$

   (b) Determine the order and degree of the following differential equation:

   $$\frac{d^4y}{dx^4}+\left(\frac{dy}{dx}\right)^3+x^2y=0$$

   (c) Find out the integrating factor of the differential equation $\dfrac{dx}{dy}+3\dfrac{x}{y}=\dfrac{1}{y^2}$.

   (d) Evaluate the following integral:

   $$\int_0^5 \cos x\,\delta(x-\pi)\,dx$$

   (e) Give an example of a scalar field.

   (f) Write down the expression of the gradient operator in spherical polar co-ordinates.

   (g) Obtain $\dfrac{1}{f(D^2)}\sin\alpha x$, where $D=\dfrac{d}{dx}$.

   (h) What is the physical significance of a scalar triple product?

GROUP-B

Answer any *three* questions from the following 5×3 = 15

2. (a) Find out the transformation matrix that describes a rotation by an angle 'θ' in the anti-clockwise direction about the 'z' axis. 3

   (b) Show that the above rotation preserves the vector dot product. 2

3. Evaluate $\iint_S \vec{r}\cdot\overrightarrow{dS}$ over the unit cube defined by the point (0, 0, 0) and unit intercepts on the positive x, y and z axis. 5

4. (a) Prove that: $\vec{\nabla}r^n = nr^{n-1}\hat{r}$ 2

   (b) Consider the function, $f(x)=x|x|$. Sketch this function for both positive and negative values of x. Also check whether $f'(x)$ exists at $x=0$. 3

5. (a) Find out the first three terms in the Taylor's expansion of $f(x) = \tan x$ about $x = \frac{\pi}{4}$. 2

(b) Find out the point on the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minima. Calculate the value of $f(x, y, z)$ at the minima. 3

6. Obtain the unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$ in terms of $\hat{i}, \hat{j}, \hat{k}$. Hence show that $\hat{r} \cdot \hat{r} = 1$ and $\hat{\theta} \cdot \hat{\phi} = 0$. 5

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. (a) Show that $\vec{\nabla} \cdot (\phi \vec{A}) = \vec{\nabla}\phi \cdot \vec{A} + \phi \vec{\nabla} \cdot \vec{A}$. 2

(b) Solve the following differential equation: 4

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^3 + x$$

(c) 2% of the items manufactured by a company are found to be defective. What is the probability that there are three defective items in a sample of 100? 4

8. (a) Show that the Dirac-Delta function can be represented as a limiting case of the Gaussian function. 3

(b) Given the coordinate transformation $u_1 = xy$, $2u_1 = x^2 + y^2$, $u_3 = z$. 2+2

(i) Show that the coordinate system is not orthogonal.

(ii) Find out ds^2 for the system.

(c) Show that $df = -(y^2 + xy)\,dx + x^2 dy$ is not an exact differential but $(xy^2)^{-1} df$ is. 3

9. (a) Find out the value of *P* that will make the following vectors coplanar. 3

$$\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}\ ,\ \vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}\ ,\ \vec{C} = \hat{i} + \hat{j} - P\hat{k}$$

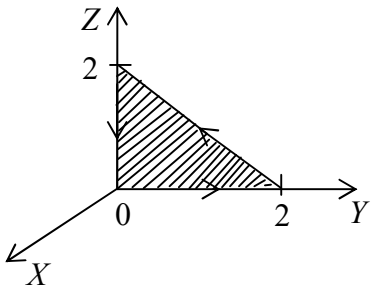
(b) Show that $\vec{V} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is a solenoidal vector. 3

(c) Express line element, area element, and volume element for an orthogonal curvilinear coordinate system. 4

10. (a) Solve the following differential equation: 3

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

(b) Verify Stoke's theorem for the function $\vec{f} = xy\,\hat{i} + 2yz\hat{j} + 3zx\hat{k}$ using the triangular shaded area of the figure below: 4



(c) If $x = r\cos\theta$ and $y = r\sin\theta$, show that $\frac{\partial(x, y)}{\partial(r, \theta)} = r$ and $\frac{\partial(r, \theta)}{\partial(x, y)} = \frac{1}{r}$. 3

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