'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2023

SEC1-P1-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains SEC1A and SEC1B. Candidates are required to answer any *one* from the *two* SEC1 courses and they should mention it clearly on the Answer Book.

SEC1A

LOGIC AND SETS

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Prove that for every positive integer n, n^2+n is always even. 3

(b) Show that the proposition $(p\wedge q)\to(p\vee q)$ is a tautology. 3

(c) Find the negation of the following statements: 3

(i) $\exists\, x\; p(x)\wedge\exists\, y\; q(y)$

(ii) $\forall\, x\; p(x)\vee\exists\, y\; q(y)$

(d) For any two cardinal numbers α and β, prove that $\alpha\beta=\beta\alpha$. 3

(e) Prove that the set $\mathbb{Q}$ of rational numbers is countable. 3

(f) State Zorn's Lemma for a poset. 3

GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) (i) Using well ordering principle of natural numbers, prove that every subset of a countable set is countable. 4

(ii) Define a bijection from $\mathbb{N}$ to $\mathbb{Z}$. 2

(b) Let p, q and r be the following statements: 2+2+2

p: Today is Friday.

q: It is raining.

r: It is hot.

Write the following compound statements in word:

(i) $(p\wedge\sim q)\to\sim r$

(ii) $\sim q\to(r\wedge p)$

(iii) $(p\vee q)\to\sim r$

(c) (i) Examine whether the following argument is logically correct: 3

"If I study then I will not fail in Mathematics. If I do not play cards then I will study. But I failed in Mathematics. Therefore, I played cards."

(ii) Prove the following logical equivalence using truth table: 3

$$\sim (p \to q) \equiv p \wedge (\sim q)$$

(d) (i) Prove that for any positive integer n, 7 divides $3^{2n+1} + 2^{n+2}$. 4

(ii) State second principle of Mathematical induction. 2

(e) Let α and β be two cardinal numbers such that $\beta \le \alpha$, where α is infinite. Prove that $\alpha + \beta = \alpha$. 6

(f) State the negation, converse and contrapositive of the following statement: 2+2+2

'Every convergent sequence of real numbers is bounded.'

GROUP-C

3. Answer any ***two*** questions: 12×2 = 24

(a) (i) Prove that the set $\mathbb{R}$ of real numbers is uncountable. 6

(ii) Let S be the set of all sequences whose elements are the digits 0 and 1. Prove that S is uncountable. 6

(b) (i) If α and β are two ordinals then prove that $\alpha \subseteq \beta$ if and only if $\alpha = \beta$ or $\alpha \in \beta$. 6

(ii) Let α, β and γ be three ordinals. Then prove that $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$. 6

(c) (i) Prove the following logical equivalences: 3+3

$$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \neg p;$$

$$(p \vee q) \wedge (\sim p \vee \sim q) \equiv (q \wedge \sim p) \vee (p \wedge \sim q)$$

(ii) Prove that the following are tautologies: 2+4

$$p \vee (\sim (p \wedge q)) \text{ and } ((p \to q) \wedge (\sim q)) \to \sim p$$

(d) (i) Prove the following equivalences: 4+4

$$\sim (\forall x \in A)\, p(x) \equiv (\exists x \in A) \sim p(x);$$

$$\sim (\exists x \forall y,\ p(x,\ y)) \equiv \forall x \exists y \sim p(x, y)$$

(ii) Determine the validity of the following argument: 4

All of my friends are musicians.
Sourav is my friend.
None of my neighbours are musicians.
Therefore, Sourav is not my neighbour.

SEC1B

Graph Theory

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Draw a graph which is both Eulerian and Hamiltonian and justify.

(b) Justify the statement: "Every tree is a bipartite graph".

(c) Find the number of spanning trees in K_5.

(d) Prove that any tree (with more than one vertex) must have at least two pendant vertices.

(e) For which n does the graph K_n contains an Euler circuit? Explain.

(f) Give three equivalent definitions of a tree.

GROUP-B

2. Answer any ***four*** questions: $6\times4=24$

(a) Prove that any graph is bipartite if and only if it does not contain any odd cycle. 6

(b) (i) Prove that the addition of any edge to a tree creates a cycle. 3

(ii) What is the maximum number of edge disjoint Hamiltonian cycles on K_6? 3

(c) Let $G=(V,E)$ be a simple graph of order n having k components. Prove that the size (edges) of G can be atmost $\frac{1}{2}(n-k)(n-kH)$.

(d) Prove that an Euler graph G will be arbitrarily traceable from a vertex v is G, if and only if every circuit in G contains v.

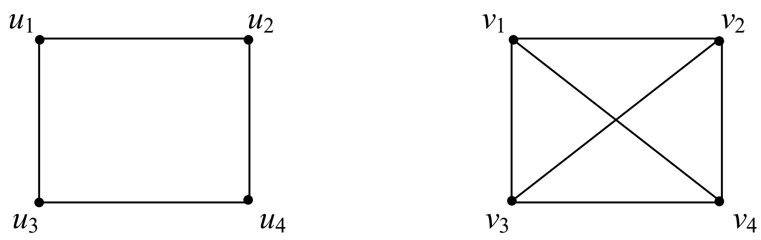
(e) Draw two graphs with degree sequence $\{3,3,3,3,4\}$. Find their adjacency matrices.

(f) Let G be a Hamiltonian graph that is not a cycle. Prove that G has atleast 2 vertices of degree greater than or equal to 3.

GROUP-C

Answer any *two* questions $12\times2=24$

3. (a) Define isomorphism of two graphs. Examine whether the following graphs are isomorphic. 4



(b) Let 'G be a simple graph of order n if $\deg(u)+\deg(v)\geq n-1$' for every two non-adjacent vertices u and v of G. Show that G is connected. 5

(c) Find the smallest positive integer n such that the complete graph k_n has atleast 400 edges. 3

4. (a) Let G be a graph of order n and size m. Let G have k-components show that G will be a forest if and only if $m-n+k=0$. 5

(b) Prove that every simple graph with $n(\geq 2)$ vertices must have at least one pair of vertices whose degrees are same. 4

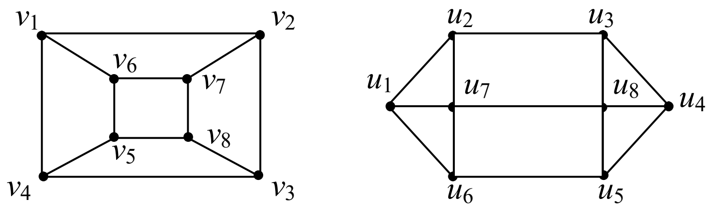
(c) Show that a k-regular graph of order $2k-1$ is Hamiltonian. 3

5. (a) Draw the graph whose incidence matrix is given by 6

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

(b) Prove that if a graph is regular of odd degree then it has even order. 3

(c) Show that the following graphs are Hamiltonian but not Eulerian. 3



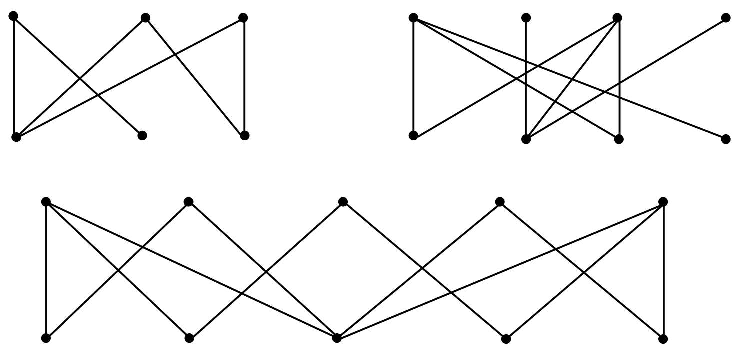
6. (a) A salesman has to visit four cities namely A, B, C, D starting from the home city A. He does not want to visit any city twice before completing his tour of all cities and would like to return to the home city A. Cost of going from one city two another are given below. 6

	A	B	C	D
A	–	5	2	3
B	2	–	4	3
C	2	4	–	7
D	4	3	7	–

Determine the optimal route and the minimum expenditure to be done by the salesman.

(b) Find n for which the complete graph k_n is (i) Semi-Eulerian (ii) Eulerian. 2

(c) Find a matching of the bipartite graphs below or explain why no matching exists. 4



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SEC1A

LOGIC AND SETS

GROUP-A

1. Answer any ***four*** questions: $3\times 4 = 12$

(a) Determine all integer solutions of the congruence $3x \equiv 7 \pmod 4$. 3

(b) If $A \cup B = B$ holds for all subsets B, prove that $A = \phi$. 3

(c) Prove the following logical equivalence: 3

$$(p \to q) \vee r \equiv (p \vee r) \to (q \vee r)$$

(d) If $n(P(P(P(A)))) = 32$, then find $n(A)$. 3

(e) If $A = \{x : 0 \le x \le 1 \text{ or } 2 \le x \le 3\}$ and $B = \{x : 0 \le x \le 2\}$ then draw the figure of the set $A \times B$ in $\mathbb{R}^2$. 3

(f) Prove or disprove: 'Every transitive relation on $\mathbb{R}$ is a reflexive relation'. 3

GROUP-B

Answer any *four* questions $6\times 4 = 24$

2. (a) If $A_n = \left(1-\frac{1}{n},\ 2+\frac{1}{n}\right]$ and $B_n = \left(1+\frac{1}{n},\ 2-\frac{1}{n}\right]$ for all $n \in \mathbb{N}$, then find $\bigcap_{n=1}^{\infty}(A_n \setminus B_n)$. 3

(b) Let ρ be a relation defined on $\mathbb{Z}$ by '$a\rho b$ iff $|a-b| \le 3$ for all $a, b \in \mathbb{Z}$'. Examine whether ρ is an equivalence relation. 3

3. (a) If A, B and C are non-empty sets then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. 3

(b) Show that $A \setminus B$ and $A \cap B$ are disjoint sets. 3

4. Let ρ be a relation on a set A. Then prove that ρ is an equivalence relation on A if and only if the following conditions hold: 6
   (i) $\Delta_A \subseteq \rho$, where $\Delta_A = \{(a, a):\ a \in A\}$,
   (ii) $\rho = \rho^{-1}$ and
   (iii) $\rho \circ \rho \subseteq \rho$.

5. Verify whether following statements are tautologies: 3+3
   (i) $p \to (q \to (p \wedge q))$
   (ii) $(p \vee q) \to (q \to (p \wedge q))$.

6. If $p,\ q$ are primitive statements, prove that 6

$$(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \leftrightarrow (p \wedge q)$$

7. (a) Test the logical validity of the following argument: 4

   All men are mortal. Sachin is a man. Therefore, Sachin is mortal.

   (b) Prove that the set of all prime numbers is an infinite set. 2

GROUP-C

Answer any *two* questions 12×2 = 24

8. (a) Let A, B and C be three sets.
   (i) If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, prove that $B = C$. 3
   (ii) If $A \Delta B = A \Delta C$, prove that $B = C$. 3

   (b) Let ρ be an equivalence relation on a set A. Prove that $\rho = \{[a]:\ a \in A\}$ is a partition of A. Here $[a]$ denotes the equivalence class of a w. r. t. ρ. 6

9. (a) Let $\mathcal{F} = \{I_n : n \in \mathbb{N}\}$, where $I_n = \left\{x \in \mathbb{R}:\ -\left(1+\frac{1}{n}\right) < x < \left(1+\frac{1}{n}\right)\right\}$. Prove that $\bigcup_{n \in \mathbb{N}} I_n = \{x \in \mathbb{R} :\ -2 < x < 2\}$ and $\bigcap_{n \in \mathbb{N}} I_n = \{x \in \mathbb{R}:\ -1 \le x \le 1\}$. 3+3

   (b) Among 60 students in a class, 36 got an A in the first examination and 31 got an A in the second examination. If 25 students did not get an A in either examination, how many students got A in both the examinations? 3

   (c) Let A, B, C be three sets. Draw Venn-diagrams of $(A \cup B) = (A \cup C)$ but $B \ne C$. 3

10. (a) Suppose a set X has 5 elements. Find $n(P(X))$ and $P(P(P(\phi)))$. Here $P(X)$ denotes the power set of X. 4

   (b) Let A, B, C be three sets. Prove that 4

$$((A \setminus B) \cup (A \cap B)) \cap ((B \setminus A) \cup (A \cup B)^c) = \phi$$

   (c) Let I_n denote the first n natural numbers. Describe the set $I_n \setminus I_m$ if (i) $n > m$ and (ii) $n = m$. 2+2

11.(a) Find the negation of the following statements: 2+2+2

(i) $\exists x\, p(x) \wedge \exists y\, q(y)$

(ii) $\forall x\, p(x) \vee \exists y\, q(y)$

(iii) $(\forall x)\, (\exists y)\, [x^2 \leq y]$

(b) Find the negation, converse and contrapositive of the following statements: 3+3

(i) If x is a real number then it is a rational number.

(ii) Every equivalence relation on a set S is a symmetric relation on S.

SEC1B

C++

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) What is friend function? Describe its importance.

(b) Write a short note on object oriented programming language.

(c) Write a loop statement that will show the following output:

```
6
6 5
6 5 4
6 5 4 3
6 5 4 3 2
6 5 4 3 2 1
```

(d) Write a C++ program that displays first 50 odd numbers.

(e) What is copy constructor? Illustrate with a suitable C++ example.

(f) Explain the use of inline function with the help of a suitable function.

GROUP-B

Answer any *four* questions 6×4 = 24

2. Write a C++ program to print all prime numbers between two positive numbers.

3. Write a C++ program that counts the number of even and odd elements in an array.

4. Write a C++ program to exchange the biggest and smallest digits of an input number.

5. How does polymorphism promote extensibility? Explain various types of polymorphism with example. 2+4

6. Write a C++ program to generate Fibonacci sequence using overloading of increment operator. 6

7. What is inheritance? What are base and derived classes? Give a suitable example for inheritance. 2+2+2

GROUP-C

Answer any *two* questions 12×2 = 24

8. (a) Differentiate between compiler time polymorphism and run time polymorphism. 6
   (b) Describe the importance of destructor. Explain its use with the help of an example. 3+3

9. (a) Explain class template. How many types of templates are there in C++? 3+3
   (b) What is the difference between error and exception? Explain what are the different types of exceptions. 3+3

10. (a) What is exception handling? Explain how to handle an exception with appropriate example. 2+4
    (b) Write a C++ program to pick up the largest number from a 5 row by a 5 column matrix. 6

11. (a) Explain the difference between class and object in object oriented programming language. 4
    (b) Explain enumeration data type with an example. 4
    (c) List out characteristics of constructors. 4

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