'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2023

GE2-P1-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE1 and MATHGE4. Candidates are required to answer any *one* from the *two* MATHGE courses and they should mention it clearly on the Answer Book.

MATHGE1

CALCULUS, GEOMETRY AND DE

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) Show that the curve $y = xe^{-x^2}$ has point of inflexion at $x = \pm\dfrac{\sqrt{3}}{2}, 0$. 3

(b) Prove that the differential equation of all circles touching the y-axis at the origin is $(y^2 - x^2)\,dx - 2xy\;dy = 0$. 3

(c) Is $(x^3 + 3y^2x)\;dx + (y^3 + 3x^2y)\;dy = 0$ an exact equation? If yes, find its primitive. 3

(d) If r_1, r_2 be two mutually perpendicular radius vector of the ellipse $r^2 = \dfrac{b^2}{1 - e^2\cos^2\theta}$, show that $\dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$. 3

(e) Find a and b in order that the $\lim\limits_{x\to0}\dfrac{a\sin 2x - b\sin x}{x^3} = 1$. 3

(f) Trace the curve $y = \dfrac{8a^3}{x^2 + 4a^2}$. 3

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4 = 24$

(a) Show that the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ is a rectangular hyperbola, reduce the equation to its canonical form. 6

(b) Find reduction formula for $\int\cos^n x\,dx$, n being a positive integer and evaluate 6

$$\int_0^{\pi/4}\cos^7 x\;dx$$

(c) If $y = e^{\tan^{-1}x}$, prove that 6

$$(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$$

(d) Solve: $y(xy+2x^2y^2)\,dx+x(xy-x^2y^2)\,dy=0$ 6

(e) Find the envelop of the straight line $\frac{x}{a}+\frac{y}{b}=1$, when $ab=c^2$ (c is a constant). 6

(f) Prove that the equation of the chord joining the points $\theta=\theta_1$, $\theta=\theta_2$ on the circle $r=2a\cos\theta$ is $r\cos(\theta-\theta_1-\theta_2)=2a\cos\theta_1\cos\theta_2$. 6

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) Solve the differential equation: $(x+2y-3)\,dx=(2x+y-3)\,dy$ 6

(ii) Solve: $x\frac{dy}{dx}+y=y^2x^3\cos x$ 6

(b) (i) Find the asymptotes of the curve 5

$$2x^3-x^2y+2xy^2+y^3-4x^2+8xy-4x+1=0$$

(ii) Find the area bounded by one arch of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ and the X-axis. 4

(iii) Find the inflection points and the intervals of concavity up and down of $f(x)=3x^2-9x+6$. 3

(c) Find the equation of the sphere:

(i) Which passes through the circle $x^2+y^2+z^2-x-2y+9z=4$; $x+2y+3z+7=0$ and touches the plane $3x+4z=16$. 6

(ii) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$, which pass through the point $(2, 3, -4)$. 6

(d) (i) Find the volume of the solid obtained by the revolving cardioide $r=a(1+\cos\theta)$ about the initial line. 6

(ii) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse 6

$$x^2+2y^2=1 \quad , \quad z=3$$

MATHGE4

Group Theory

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Prove that $(ab)^2=a^2b^2$ in a group G, then $ab=ba$, $\forall\, a, b\in G$. 3

(b) Let G be an abelian group of order 6 containing an element of order 3. Then prove that G is cyclic group. 3

(c) Suppose G is the group of all non-zero real numbers under multiplication and $f:G\to G$ be defined by $f(x)=1$ if $x>0$, and $f(x)=-1$ if $x<0$. Show that f is a homomorphism. 3

(d) Find the generators of the group $(\mathbb{Z}_{16}, +)$. 3

(e) Every abelian group is cyclic or not? Explain your answer. 3

(f) Prove that every proper subgroup of a group of order 6 is cyclic. 3

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4 = 24$

(a) Prove that every group of order less than 6 is commutative. 6

(b) State and prove Lagrange's theorem. 6

(c) Let $(G, \circ)$ be a semigroup and for any two elements a, b in G, each of the equations $a \circ x = b$ and $y \circ a = b$ has a solution in G. Then $(G, \circ)$ is a group. 6

(d) Let $(G, \circ)$ and $(G', *)$ be two groups and $\phi : G \to G'$ be an epimorphism. Then 6
(i) If G is commutative, then G' is commutative but the converse is not true.
(ii) If G is cyclic, then G' is cyclic, but converse is not true.

(e) If every cyclic subgroup of a group G is normal in G, then prove that every subgroup of G is normal in G. 6

(f) Prove that the set S_n, set of all permutations on a set of n-distinct elements form a non-abelian group with respect to multiplication of permutations. 6

GROUP-C

3. Answer any ***two*** questions from the following: $12\times2 = 24$

(a) (i) Let $\phi : (G, \circ) \to (G', *)$ be a homomorphism. Define kernel of ϕ. Prove that ϕ is one-to-one if and only if $\ker\phi = \{e_G\}$. 6

(ii) Show that two finite cyclic groups of same order are isomorphic. 6

(b) (i) Let $G = (\mathbb{R}, +)$, $G' = (\{z \in \mathbb{C} : |z| = 1\}, \circ)$ and $\phi : G \to G'$ is defined by $\phi(x) = \cos 2\pi x + i \sin 2\pi x$, $x \in \mathbb{R}$. Prove that ϕ is a homomorphism. Determine $\ker\phi$. 6

(ii) If H is a subgroup of a cyclic group G, then show that the quotient group G/H is cyclic. 6

(c) (i) Prove that the symmetric group S_3 has a trivial centre. 6

(ii) Let H be subgroup of a group G. Then H is normal in G if and only if $h \in H$ and $x \in G \Rightarrow xhx^{-1} \in G$. 6

(d) (i) In a group $(G, \circ)$, the elements a and b commute and $o(a), o(b)$ are prime to each other. Then show that $o(a \circ b) = o(a) \circ o(b)$. 6

(ii) Let H and K be finite subgroups of G, such that HK is subgroup of G. Then 6

$$o(HK) = \frac{o(H)\, o(K)}{o(H \cap K)}$$

——×——

'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2023

GE2-P1-MATHEMATICS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE1, MATHGE2, MATHGE3, MATHGE4 and MATHGE5. Candidates are required to answer any *one* from the *five* MATHGE courses and they should mention it clearly on the Answer Book.

MATHGE1

CALCULUS, GEOMETRY AND DE

GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4=12$

(a) Show that the curve $y=xe^{-x^2}$ has point of inflexion at $x=\pm\dfrac{\sqrt{3}}{2}, 0$. 3

(b) Prove that the differential equation of all circles touching the y-axis at the origin is $(y^2-x^2)\,dx-2xy\;dy=0$. 3

(c) Is $(x^3+3y^2x)\;dx+(y^3+3x^2y)\;dy=0$ an exact equation? If yes, find its primitive. 3

(d) If r_1, r_2 be two mutually perpendicular radius vector of the ellipse $r^2=\dfrac{b^2}{1-e^2\cos^2\theta}$, show that $\dfrac{1}{r_1^2}+\dfrac{1}{r_2^2}=\dfrac{1}{a^2}+\dfrac{1}{b^2}$. 3

(e) Find a and b in order that the $\lim\limits_{x\to 0}\dfrac{a\sin 2x-b\sin x}{x^3}=1$. 3

(f) Trace the curve $y=\dfrac{8a^3}{x^2+4a^2}$. 3

GROUP-B

2. Answer any ***four*** questions from the following: $6\times 4=24$

(a) Show that the conic represented by $3x^2-8xy-3y^2+10x-13y+8=0$ is a rectangular hyperbola, reduce the equation to its canonical form. 6

(b) Find reduction formula for $\int\cos^n x\,dx$, n being a positive integer and evaluate 6

$$\int_0^{\pi/4}\cos^7 x\;dx$$

(c) If $y = e^{\tan^{-1} x}$, prove that 6

$$(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$$

(d) Solve: $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$ 6

(e) Find the envelop of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, when $ab = c^2$ (c is a constant). 6

(f) Prove that the equation of the chord joining the points $\theta = \theta_1$, $\theta = \theta_2$ on the circle $r = 2a\cos\theta$ is $r\cos(\theta - \theta_1 - \theta_2) = 2a\cos\theta_1\cos\theta_2$. 6

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) Solve the differential equation: $(x + 2y - 3)\,dx = (2x + y - 3)\,dy$ 6

(ii) Solve: $x\frac{dy}{dx} + y = y^2x^3\cos x$ 6

(b) (i) Find the asymptotes of the curve 5

$$2x^3 - x^2y + 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$$

(ii) Find the area bounded by one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and the X-axis. 4

(iii) Find the inflection points and the intervals of concavity up and down of $f(x) = 3x^2 - 9x + 6$. 3

(c) Find the equation of the sphere:

(i) Which passes through the circle $x^2 + y^2 + z^2 - x - 2y + 9z = 4$; $x + 2y + 3z + 7 = 0$ and touches the plane $3x + 4z = 16$. 6

(ii) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$, which pass through the point $(2, 3, -4)$. 6

(d) (i) Find the volume of the solid obtained by the revolving cardioide $r = a(1 + \cos\theta)$ about the initial line. 6

(ii) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse 6

$$x^2 + 2y^2 = 1 \quad , \quad z = 3$$

MATHGE2

ALGEBRA

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Verify Cayley Hamilton theorem for the square matrix $\begin{pmatrix} 2 & 1 \\ 0 & 5 \end{pmatrix}$. 3

(b) Apply Descartes' rule of signs to find the nature of the roots of the equation 3

$$2x^4 + 2x^3 - 3x^2 - 2x + 1 = 0$$

(c) Find the quadratic equation whose roots are twice the roots of $2x^2-5x+2=0$. 3

(d) Prove that the greatest value of a^2b^3 is $\frac{3}{16}$, where a and b are positive satisfying the condition $3a+4b=5$. 3

(e) If sum of the roots of the equation $x^3+px^2+qx+r=0$ be zero, prove that $r=pq$. 3

(f) Find the eigenvalues of the matrix $A=\begin{pmatrix}5 & 4\\ 1 & 2\end{pmatrix}$. 3

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) Prove that $3^{4n+2}+5^{2n+1}$ is divisible by 14 for all $n\in\mathbb{N}$. 6

(b) If α, β, γ be the roots of equation $px^3+3qx^2+3rx+s=0$ find the values of 3+3

(i) $\sum\alpha^3$ (ii) $\sum\alpha^2\beta$

(c) (i) Find all solutions of the congruence $19x\equiv 8 \pmod{32}$. 3+3

(ii) Find the integer in the unit place of 7^{316}.

(d) Examine whether the following system of equations has a solution or not: 6

$$\begin{aligned} x_1+x_2+x_3&=1\\ 2x_1+x_2+2x_3&=2\\ 3x_1+2x_2+3x_3&=5 \end{aligned}$$

If $\left.\begin{aligned} x_1+x_2+x_3&=0\\ 2x_1+x_2+2x_3&=0\\ 3x_1+2x_2+3x_3&=0 \end{aligned}\right\}$ Then what can we say about its solution.

(e) Remove the second term of the equation $x^3+6x^2+12x-19=0$ and solve it. 6

(f) Find the eigenvalues and the corresponding eigenvectors of the matrix. 6

$$\begin{pmatrix}1 & -1 & 2\\ 2 & -2 & 4\\ 3 & -3 & 6\end{pmatrix}$$

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) If $\sin(\alpha+i\beta)=x+iy$, then prove that 6

$$\frac{x^2}{\sin^2\alpha}-\frac{y^2}{\cos^2\alpha}=1 \quad\text{and}\quad \frac{x^2}{\cosh^2\beta}+\frac{y^2}{\sinh^2\beta}=1$$

(ii) If a, b, c are all positive real numbers, prove that 6

$$\frac{a^2+b^2}{a+b}+\frac{b^2+c^2}{b+c}+\frac{c^2+a^2}{c+a}\geq(a+b+c)$$

(b) (i) If α, β, γ are the roots of the equation $x^3+qx+r=0$, find the equation whose roots are $\beta+\gamma-2\alpha$, $\gamma+\alpha-2\beta$, $\alpha+\beta-2\gamma$. 6

(ii) Reduce the matrix A to row reduced echelon form and hence find its rank, where $A=\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 3 \\ 1 & -2 & 7 & 5 \end{pmatrix}$. 6

(c) (i) Determine the conditions for which the system of equations 6

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

admits of only one solution, no solution and many solutions.

(ii) Prove by induction, that $1\cdot 2+2\cdot 2^2+3\cdot 2^3+\cdots\cdots+n\cdot 2^n=(n-1)2^{n+1}+2$ for all $n\in\mathbb{N}$. 6

(d) (i) Solve the equation by Cardon's method: $x^3-18x-35=0$ 6

(ii) Expand $\cos^7\theta$ in a series of cosines of multiples of θ. 3

(iii) Find amplitude of $\dfrac{(1-i)\,(2+3i)}{(i+1)\,(2-3i)}$. 3

MATHGE3

DE AND VECTOR CALCULUS

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Find $\dfrac{1}{D^2-1}(4xe^x)$. 3

(b) Find the ordinary and singular points of the following differential equation: 3

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

(c) Verify if the functions x^2, x^3 and x^{-2} are linearly independent. 3

(d) Prove that the three vectors $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$ and $\vec{c}\times(\vec{a}\times\vec{b})$ lie on the same plane. 3

(e) Check if $\vec{f}(t)=t\hat{i}+|t|\,\hat{j}+2\hat{k}$ is differentiable at $t=0$. 3

(f) Find $\lim\limits_{t\to 0}\left(t\sin\frac{1}{t}\,\hat{i}+t^2\hat{j}+t\cos\frac{1}{t^2}\hat{k}\right)$. 3

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) Solve by the method of variation of parameters: 6

$$\frac{d^2y}{dx^2}+a^2y=\sec ax$$

(b) Solve: $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=20\left(x+\dfrac{1}{x}\right)$ 6

(c) Find the power series solution of the differential equation 6

$$(x^2-1)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0 \text{ about the point } x=0$$

(d) Solve: $(D+1)x-Dy=\sin t$; $Dx+y=\cos t$ 6

$\left(\text{when } D\equiv\dfrac{d}{dt}\right)$, given that $x=1$, $y=0$ for $t=0$.

(e) If $\vec{r}_1=3t^2\,\hat{i}+(2t-1)\,\hat{j}+5t\,\hat{k}$, $\vec{r}_2=2t\,\hat{i}+\hat{j}-4t^2\,\hat{k}$ and $\vec{r}_3=2(t^2+1)\,\hat{i}-(t-1)\,\hat{j}+2t\,\hat{k}$, 6

then find $\displaystyle\int_0^3 \vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2)\,dt$.

(f) If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, prove that any vector $\vec{d}$ can be represented as 6

$$\vec{d}=\frac{[\vec{d}\;\vec{b}\;\vec{c}]}{[\vec{a}\;\vec{b}\;\vec{c}]}\vec{a}+\frac{[\vec{d}\;\vec{c}\;\vec{a}]}{[\vec{a}\;\vec{b}\;\vec{c}]}\vec{b}+\frac{[\vec{d}\;\vec{a}\;\vec{b}]}{[\vec{a}\;\vec{b}\;\vec{c}]}\vec{c}$$

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) Solve: $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=e^{-x}\sin x$ 6

(ii) Find the general solution of the differential equation 6

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=x^2e^x$$

by the method of undetermined coefficients.

(b) (i) Solve the following system of linear differential equation using the operator $D\equiv\dfrac{d}{dt}$: 6

$$2\frac{dx}{dt}+\frac{dy}{dt}-x-y=1 \quad ; \quad \frac{dx}{dt}+\frac{dy}{dt}+2x-y=t$$

(ii) Find the volume of the tetrahedron whose vertices are $(2,-4,5)$, $(1,3,-2)$, $(3,-1,-4)$ and $(-5,2,-1)$. 6

(c) (i) Verify that x is a solution of the reduced equation of 6

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=x^2$$

Solve the equation after reducing it to a 1st order linear equation.

(ii) Find the line integral $\displaystyle\int_\Gamma (y^2+z^2)\,\hat{i}+(z^2+x^2)\,\hat{j}+(x^2+y^2)\,\hat{k}$ over the straight line Γ joining the points $(0,0,0)$ and $(2,1,1)$. 6

(d) (i) Find $\vec{r}=\vec{r}(t)$ if $\dfrac{d^2\vec{r}}{dt^2}=6t\,\hat{i}-24t^2\,\hat{j}+4\sin t\,\hat{k}$ under the given conditions $\vec{r}=2\,\hat{i}+\hat{j}$ and $\dfrac{d\vec{r}}{dt}=-\hat{i}-3\hat{k}$ at $t=0$. 6

(ii) Find the shortest distance between two lines through $(6, 2, 3)$ and $(4, 0, -3)$ and parallel to the vectors $(1, -2, 3)$ and $(3, -1, 4)$ respectively. 6

MATHGE4

GROUP THEORY

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) Prove that $(ab)^2=a^2b^2$ in a group G, then $ab=ba$, $\forall\, a, b\in G$. 3

(b) Let G be an abelian group of order 6 containing an element of order 3. Then prove that G is cyclic group. 3

(c) Suppose G is the group of all non-zero real numbers under multiplication and $f: G\to G$ be defined by $f(x)=1$ if $x>0$, and $f(x)=-1$ if $x<0$. Show that f is a homomorphism. 3

(d) Find the generators of the group $(\mathbb{Z}_{16}, +)$. 3

(e) Every abelian group is cyclic or not? Explain your answer. 3

(f) Prove that every proper subgroup of a group of order 6 is cyclic. 3

GROUP-B

2. Answer any ***four*** questions from the following: $6\times4=24$

(a) Prove that every group of order less than 6 is commutative. 6

(b) Sate and prove Lagrange's theorem. 6

(c) Let $(G, \circ)$ be a semigroup and for any two elements a, b in G, each of the equations $a\circ x=b$ and $y\circ a=b$ has a solution in G. Then $(G, \circ)$ is a group. 6

(d) Let $(G, \circ)$ and $(G', *)$ be two groups and $\phi: G\to G'$ be an epimorphism. Then 6

(i) If G is commutative, then G' is commutative but the converse is not true.

(ii) If G is cyclic, then G' is cyclic, but converse is not true.

(e) If every cyclic subgroup of a group G is normal in G, then prove that every subgroup of G is normal in G. 6

(f) Prove that the set S_n, set of all permutations on a set of n-distinct elements form a non-abelian group with respect to multiplication of permutations. 6

GROUP-C

3. Answer any ***two*** questions from the following: $12\times2=24$

(a) (i) Let $\phi:(G, \circ)\to(G', *)$ be a homomorphism. Define kernel of ϕ. Prove that ϕ is one-to-one if and only if $\ker\phi=\{e_G\}$. 6

(ii) Show that two finite cyclic groups of same order are isomorphic. 6

(b) (i) Let $G=(\mathbb{R},\ +)$, $G'=(\{z\in\mathbb{C} : |z|=1\},\ \circ)$ and $\phi : G \to G'$ is defined by $\phi(x)=\cos 2\pi x + i\sin 2\pi x$, $x\in\mathbb{R}$. Prove that ϕ is a homomorphism. Determine $\ker\phi$. 6

(ii) If H is a subgroup of a cyclic group G, then show that the quotient group G/H is cyclic. 6

(c) (i) Prove that the symmetric group S_3 has a trivial centre. 6

(ii) Let H be subgroup of a group G. Then H is normal in G if and only if $h\in H$ and $x\in G \Rightarrow xhx^{-1}\in G$. 6

(d) (i) In a group $(G,\ \circ)$, the elements a and b commute and $o(a), o(b)$ are prime to each other. Then show that $o(a\circ b)=o(a)\circ o(b)$. 6

(ii) Let H and K be finite subgroups of G, such that HK is subgroup of G. Then 6

$$o(HK)=\frac{o(H)\,o(K)}{o(H\cap K)}$$

MATHGE5

NUMERICAL METHODS

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Prove that $\frac{\Delta^2}{E}(x^3)=6xh^2$, where the notations are used have their usual meaning. 3

(b) Find the function whose first difference is x^3-4x^2+3. 3

(c) Show that Bisection method converges linearly. 3

(d) Show that the equation $x^2+\log x=0$ has exactly one root lies in $\left[\frac{1}{3},\ 1\right]$. 3

(e) Define the 'degree of precession' of a Quadrature formula for numerical integration. What is the degree of precession of Trapezoidal rule? 3

(f) Use modified Euler method to compute $y(0.1)$ for the following initial value problem: 3

$$\frac{dy}{dx}=x+y \quad , \quad y(0)=1 \quad ; \quad \text{taking } h=0.1$$

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) Using Newton-Raphson method find a real root of $3x-\cos x-1=0$. 6

(b) Define shift operator E. Prove that $E=e^{hD}$, where h is the step length and $D=\frac{d}{dx}$. 6

(c) Solve the following system of linear equations by Gauss Seidel method: 6

$$20x+5y-2z=14$$
$$3x+10y+z=17$$
$$x-4y+10z=23$$

(d) What is interpolation? Establish Lagrange's polynomial interpolation formula. 6

(e) Evaluate $\int_{0.1}^{0.7}(e^x+2x)\,dx$, by Simpson's $\frac{1}{3}$rd rule, taking $h=0.1$ and correct upto 3 decimal places. 6

(f) Use Euler's method to solve the following problem for $x=0.06$ by taking $h=0.02$ 6

$$\frac{dy}{dx}=\frac{y-x}{y+x} \quad \text{with} \quad y(0)=1$$

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) Evaluate the missing term in the following table: 6

x	0	1	2	3	4	5
$f(x)$	0	–	8	15	–	35

(ii) Deduce an expression for the remainder in polynomial interpolation of a function $f(x)$ with nodes $x_0,\, x_1,\, x_2,\cdots\cdots,\, x_n$. 6

(b) (i) Evaluate $\int_{1}^{4.5}\frac{e^x}{1+x}\,dx$, by Trapezoidal rule correct upto 2 decimal places taking 8 points. 6

(ii) Establish Newton's backward interpolation formula. 6

(c) (i) Use R-K Method of order 2 to solve the differential equation $\frac{dy}{dx}=x^2+y/2$ at $x=0.5$ correct to two decimal places, given that $y=1$ when $x=0$. 6

(ii) Use Gauss-Elimination method to solve the following: 6

$$-10x+6y+3z+100=0,$$
$$6x-5y+5z+100=0,$$
$$3x+6y-10z+100=0$$

Correct upto three significant figures.

(d) (i) If the third order differences of a function $f(x)$ are constants and 6

$$\int_{-1}^{1}f(x)\,dx=k\left[f(0)+f\left(\frac{1}{k}\right)+f\left(-\frac{1}{k}\right)\right],$$

then find the value of k.

(ii) Compute the root of the following by Regula-Falsi method 6

$$2x-3\sin x-5=0$$

Correct upto three decimal places.

——×——