'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 3rd Semester Examination, 2023

CC5-PHYSICS

MATHEMATICAL PHYSICS-II

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: 1×5 = 5

(a) State Dirichlet's conditions.

(b) What is cyclic coordinate?

(c) What is regular singular point?

(d) Evaluate: $\beta(3, 5)-\beta(5, 3)$

(e) What do you mean by normal modes?

(f) Solve: $\dfrac{\partial^2\theta}{\partial x\,\partial t}=0$

(g) Give two properties of spherical harmonics.

(h) Write down the Rodrigue's formula for Legendre's polynomial $P_n(x)$.

GROUP-B

Answer any *three* questions from the following 5×3 = 15

2. (a) Show that $\Gamma\frac{1}{2}=\sqrt{\pi}$. 2+(1+2)

(b) Write down the generating function for Legendre's polynomial $P_n(x)$. Hence, show that $(2n+1)xP_n(x)=(n+1)P_{n+1}(x)+nP_{n-1}(x)$.

3. (a) Express the Laplace's equation in spherical polar coordinates. 2+3

(b) Show that generalised momentum is conserved only when the corresponding generalised coordinate is cyclic.

4. Show that the shortest distance between two points in a flat space is a straight line joining them. 5

5. Solve $\dfrac{\partial u}{\partial x}=2\dfrac{\partial u}{\partial t}+u$ using method of separation of variables. 5

Given: $u(0, t)=2e^{-2t}$.

6. (a) Prove: $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$ — 3+2

(b) Reduce the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (m^2x^2 - n^2)y = 0$ in Bessel's form using $z = mx$.

GROUP-C

Answer any *two* questions from the following — 10×2 = 20

7. (a) If $x = r\cos\theta$, $y = r\sin\theta$ then prove (i) $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$, (ii) $\frac{\partial \theta}{\partial x} = \frac{1}{r^2}\frac{\partial x}{\partial \theta}$. — 3+3+2+2

(b) Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ with $u(0, y) = 2e^{-3y}$ by method of separation of variables.

(c) Can you define Gamma function $\Gamma n = \int_0^\infty e^{-x}x^{n-1}\,dx$ for negative values of *n*, i.e., $n < 0$. Justify.

(d) Express $x^3 - x^2 + 1$ in terms of Legendre polynomial.

8. (a) State and prove Parseval's theorem. — 4+6

(b) If $f(x) = -1$ for $-\pi < x < 0$

$= 1$ for $0 \le x \le \pi$

then find the Fourier series for the function.

9. (a) Show that Bessel function $J_n(x)$ is an even function when *n* is even and is an odd function when *n* is odd. — 4+6

(b) Solve completely the equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ of a vibrating string of length *l*, fixed at both ends, given that $y(0, t) = 0$, $y(l, t) = 0$, $y(x, 0) = f(x)$ and $\frac{\partial}{\partial t}\{y(x, 0)\} = 0$, $0 < x < l$.

10.(a) Discuss the singularity of the following differential equation — (2+3)+(4+1)

$$\frac{d^2y}{dx^2} + \frac{y}{x^3} = 0$$

(b) Derive the relation between Beta and Gamma function

$$\beta(m, n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$$

(c) Find the Fourier series representation of the function $f(x)$ in the interval $-\pi < x < \pi$ where,

$f(x) = -k$ when $-\pi < x < 0$

$= k$ when $0 < x < \pi$

Hence show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\cdots$.

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