'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2023

CC12-MATHEMATICS

NUMERICAL METHODS

(REVISED SYLLABUS 2023 / OLD SYLLABUS 2018)

Time Allotted: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions: 1×5 = 5

(a) Prove that $\nabla^k y_n = \Delta^k y_{n-k}$.

(b) Find the number of significant figures in V_A w.r.t V_T, where $V_A = 0.05411$, $V_T = 0.05418$.

(c) Why Newton-Raphson is called the method of tangent?

(d) State the condition of convergence of Gauss-Seidel iteration method for solving numerically system of linear equations.

(e) When a quadrature formula is called open type or closed type?

(f) Let h be the length of spacing and $(\Delta_h - \nabla_h)\, x^2 = 8$. Find h.

(g) Find the value of k for which the Trapezoidal rule with single interval [0, 1] will be exact for approximating the integral

$$\int_0^1 (x^4 - kx^3)\, dx$$

(h) Write down the iterative formula of Runge-Kutta method of order 4 stating clearly the terms involved.

GROUP-B

2. Answer any ***three*** questions: 5×3 = 15

(a) Find the inverse of the matrix 5

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

Using LU decomposition method taking $u_{11} = u_{22} = u_{33} = 1$, where $U = (u_{ij})_{3\times 3}$.

(b) Show that $f(E)\, a^x = a^x f(a)$ where $f(E)$ is a polynomial in E taking unity as the interval of differencing. 5

(c) (i) Find the missing term in the following table: 3

x	0	1	2	3	4	5
$f(x)$	0	—	8	15	—	35

(ii) What is interpolation? 2

(d) Use Runge-Kutta method of order 4 to approximate y when $x = 0.1$ and $x = 0.2$ given that 5

$$\frac{dy}{dx} = \frac{y - x}{1 + y + x}, \quad y(0) = 2$$

(e) Describe Regula-Falsi method to find a simple root of an equation $f(x) = 0$. Why this method is called an inverse linear interpolation method? 3+2

GROUP-C

3. Answer any ***two*** questions: 10×2 = 20

(a) (i) Find an approximate real root of the equation $2x + 3\sin x - 5 = 0$ correct upto 3 decimal places using secant method. 5

(ii) Show that Gauss-Seidel iteration is convergent if the system of equation is strictly diagonally dominant. 5

(b) (i) Evaluate $\int_0^{\pi/2} \sqrt{1 - 0.162\sin^2 x}\, dx$ by Simpson's $1/3^{rd}$ rule, correct upto four decimal places taking six subintervals. 5

(ii) Define degree of precision of a quadrature formula. Prove that the degree of precision of a quadrature formula of the form 1+4

$$\int_a^b f(x)\, dx = \sum_{k=0}^{n} w_k f(x_k)$$

cannot exceed $2n+1$ where x_k's are the $(n+1)$ nodes in $[a, b]$ and w_i's are $n+1$ weights given to the $(n+1)$ function values $f(x_k)$.

(c) (i) State the assumption for the applicability of Power method to determine a dominant eigen pair of a square matrix. How this method is applicable to compute the least eigen value and corresponding eigen vector of a square matrix. 1+4

(ii) Show that 3+2

$$\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\} \quad \text{and} \quad \nabla\Delta\, f(x) = (\Delta - \nabla)\, f(x)$$

where the symbols have their usual meaning.

(d) (i) Solve the following system of linear equations: 5

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 3z = 5$$

using Gauss-Jordan method, correct upto 3D.

(ii) Deduce numerical differentiation formula from Newton's backward interpolation formula without error term keeping at least three terms. 5

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