'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2023

DSE-P2-MATHEMATICS

(REVISED SYLLABUS 2023 / OLD SYLLABUS 2018)

Time Allotted: 2 Hours Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE2A and DSE2B. Candidates are required to answer any *one* from the *two* DSE2 courses and they should mention it clearly on the Answer Book.

DSE2A

NUMBER THEORY

GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) If a has order a mod p, where p is an odd prime, show that $a^k \equiv -1 \pmod{p}$. 3

(b) Which of the following Diophantine equations cannot be solved: 1+1+1

(i) $6x+4y=91$

(ii) $621x+736y=46$

(iii) $158x-57y=7$

(c) If p be any prime and a be a integer such that $\gcd(a, p)=1$, prove that following relation of Legendre symbols: 3

$$\left(\frac{ab}{p}\right)=\left(\frac{a}{p}\right)\left(\frac{b}{p}\right)$$

(d) Verify: (i) 3 is a primitive root of 7 2+1

(ii) 3 is a primitive root of 6

(e) Solve: $3x \equiv 7 \pmod{4}$ 3

(f) Find $\gcd(567, -315)$. 3

GROUP-B

2. Answer any ***four*** questions: $6\times4 = 24$

(a) If p and q are two distinct odd primes, show that 6

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right)=(-1)^{\frac{p-1}{2}}\;(-1)^{\frac{q-1}{2}}$$

(b) If p be a prime, show that $(p-1)! \equiv p-1 \pmod{(1+2+\cdots+(p-1))}$. 6

(c) If a, b, c are integers and a, b are not both zero, then show that the equation $ax+by=c$ has an integral solution iff d is a divisor of c, where $d=\gcd(a,b)$. Also if (x_0, y_0) be any particular solution, then show that all integral solutions are given by $\left(x_0+\frac{b}{d}t,\ \ y_0-\frac{a}{d}t\right)$, where $t\in\mathbb{Z}$. 6

(d) Show that $(1^3+2^3+3^3+\cdots\cdots+99^3)\times(1^5+2^5+\cdots\cdots+100^5)$ is divisible by 15. 6

(e) Consider the polynomial 6

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots\cdots+a_1x+a_0$$

with integral coefficients and $a_n\not\equiv 0 \pmod p$, where p is a prime. Prove that the congruence $f(x)\equiv 0 \pmod p$ has atmost n incongruent solutions $\pmod p$.

(f) Prove that for a Pythagorean primitive triple (x, y, z), $12/xyz$. Hence prove that $60/xyz$. 4+2

GROUP-C

3. Answer any ***two*** questions: 12×2 = 24

(a) (i) If p be an odd prime, show that there are an equal number of quadratic residues and quadratic non-residues of p. 6+6

(ii) Evaluate the values of $\left(\frac{11}{23}\right)$ and $\left(\frac{6}{31}\right)$.

(b) (i) State Wilson's theorem. Is the converse true? Justify. (2+4)+6

(ii) Show that $28\,!+233\equiv 0 \pmod{899}$.

(c) (i) State and prove Chinese Remainder theorem. 6+6

(ii) Solve: $x\equiv 3 \pmod 6$

$x\equiv 5 \pmod 7$

$x\equiv 2 \pmod{11}$

(d) (i) Prove that 2^k has no primitive roots $\forall k\geq 3$. 6+6

(ii) Let p be an odd prime. Prove that the quadratic congruence $x^2+1\equiv 0 \pmod p$ has a solution iff $p\equiv 1 \pmod 4$.

DSE2B

MECHANICS

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) What are the forces that can be omitted from the equation of virtual work?

(b) Find the centre of gravity of a circular area when the density varies as square of the distance from the diameter.

(c) Find the minimum time of oscillation of a given compound pendulum.

(d) Find the moment of inertia of the solid cone about its axes.

(e) State energy test of stability.

(f) An artificial satellite goes round the earth in 90 minutes in a circular orbit. Calculate the height of the satellite above the earth, taking the earth to be a sphere of radius 6370 km and g at the orbit of the satellite to be 980 cm/sec^2.

GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) A particle is projected in a medium whose resistance is proportional to the cube of the velocity and no other forces act on the particle. While the velocity diminishes from v_1 to v_2, the particle traverses a distance d in time t, show that

$$\frac{d}{t}=\frac{2v_1v_2}{v_1+v_2}$$

(b) Find the condition that a given system of forces can be combined into a single force.

(c) Show that the Kinetic Energy of a body of mass M moving in two dimensions is given by

$$\frac{1}{2}Mv^2+\frac{1}{2}Mk^2\dot{\theta}^2$$

where k is the radius of gyration of the body about a line through center of inertia and perpendicular to the motion.

(d) A uniform rod OA, of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about a vertical OZ, through 0 and is inclined at a constant angle α to OZ. Show that the value of α is either 0 or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$.

(e) Describe the motion of a particle under a force which is always directed towards a fixed point and varies inversely as the square of the distance from that point.

(f) If each of a system of coplanar forces be replaced by three forces acting along the sides of a triangle ABC in the plane of the forces, of type p_iBC, q_iCA and r_iAB. Show that the necessary and sufficient conditions that the system reduces to a couple are $\sum_i p_i=\sum_i q_i=\sum_i r_i$.

GROUP-C

3. Answer any ***two*** questions: 12×2 = 24

(a) (i) A string of length 'a' forms the shorter diagonal of a rhombus formed by four uniform rods, each of length 'b' and weight 'w', which are hanged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is $\frac{2w(2b-a^2)}{b\sqrt{4b^2-a^2}}$. 6+6

(ii) Four forces, each of magnitude P act on a rigid body, three of the forces act along the rectangular Cartesian Co-ordinate axis of x, y, z while the fourth force acts along the line

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

Find the equation of the central axis of the system.

(b) (i) A particle is projected with velocity V along a smooth horizontal plane on a medium whose resistance per unit mass is μ times the cube of the velocity. Show that the velocity at a time t is $\dfrac{V}{\sqrt{1+2\mu V^2 t}}$. 6+6

(ii) Find the co-ordinates of the centre of gravity of the arc of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ which lies in the positive quadrant.

(c) (i) A plank of mass M and length $2a$, is initially at rest along a line of greatest slope on a smooth plane, inclined at an angle α to the horizon and a man of mass M', starting from the upper end walks down the plank so that it does not move. Show that he will reach the other end in time $\sqrt{\dfrac{2M'a}{(M+M')\,g\sin\alpha}}$, where a is the length of the plank. 6+6

(ii) If T be the time period of a satellite circling round the earth at a distance R from the earth's centre, then prove that $r=\sqrt[3]{\dfrac{gRT^2}{4\pi^2}}$, where g = acceleration due to gravity on the earth's surface and R = the radius of the earth.

(d) (i) A triangular lamina ABC oscillates about the perpendicular from A on BC, the perpendicular being horizontal. Find the length of the simple equivalent pendulum. 6+6

(ii) State and prove Principle of Conservation of Energy.

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