'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2023

CC11-PHYSICS

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: $1\times5 = 5$

(a) Calculate the uncertainty in momentum of an electron if the uncertainty in its position is 0.4 nm.

(b) Only Hermitian operators are associated with physical observables. Why?

(c) For a particle of mass m in one dimensional harmonic oscillator potential of the form $V(x)=\frac{1}{2}m\omega^2x^2$, the first excited eigen state is $\psi(x)\sim xe^{-ax^2}$. Write the value of a.

(d) If A and B are two operators, then show that $[A,\ B^{-1}]=-B^{-1}[A,\ B]B^{-1}$.

(e) Find the probability current density corresponding to $\psi = Ae^{-\alpha x}$, where $\alpha = \phi$ a real constant and A = a complex constant.

(f) What is the Born's interpretation of a wave-function?

(g) Why is ${}^4\mathrm{D}_{1/2}$ term not split in a magnetic field? Explain.

(h) If $\psi(x)=\sin 3x$ is an eigenfunction of the operator $\frac{d^2}{dx^2}$, then find out its corresponding eigenvalue.

GROUP-B

Answer any *three* questions from the following $5\times3 = 15$

2. (a) What is the wavefunction for a free particle? Explain why this wavefunction cannot describe a localised particle. 2+3

(b) Normalise the wavefunction $\psi(x)=\dfrac{A}{\sqrt{x^2+\alpha^2}}$ in the region $-\infty < x < \infty$.

3. (a) Show that if $\hat{A}$, $\hat{B}$ are hermitian then $i[A,\ B]$ is hermitian. 2+3

(b) The wavefunction of a particle is given as $Nxe^{-x^2/2\sigma^2}$ where N is the normalising constant and σ is a constant. Where is the most probable location of the particle?

4. Define Larmor frequency. Calculate the Lande's g-factor for the $^2P_{3/2}$ state. Distinguish between LS and JJ coupling scheme S. 1+2+2

5. Consider an infinitely deep one dimensional zero potential well of width 'a' in which a particle of mass 'm' has orthonormalised energy eigenstates $\psi_1(x), \psi_2(x), \cdots\cdots$ with energies $E_1, E_2, \cdots\cdots$ respectively. Suppose particle exists at $t=0$ in the state given by 1+1+1+2

$$\psi(x, 0) = \frac{1}{\sqrt{6}}\psi_1(x) + \frac{i}{\sqrt{2}}\psi_2(x) + \frac{1}{\sqrt{3}}\psi_3(x)$$

(i) What is the probability of obtaining the energy-values E_1, E_2, E_3?

(ii) How will such a state evolve with time?

(iii) Find average value of energy.

(iv) Discuss whether $E = \dfrac{49\pi^2\hbar^2}{200ma^2}$ can be an allowed energy value?

6. Calculate the expectation value of r for the ground state (1s-state) of a hydrogen atom. The unnormalised ground state wavefunction is e^{-r/a_0}, where a_0 is the Bohr radius.

GROUP-C

Answer any *two* questions from the following $10\times2 = 20$

7. A particle of mass m moving along a line with energy E is incident from left on a step potential $V(x)$,

$$V(x) = 0 \quad \text{for} \quad x<0$$
$$= V_0 \quad \text{for} \quad x>0,$$

V_0 being a constant.

(a) Write down the time independent Schrödinger equation for the system. 2

(b) Obtain the solution of the wave equation for (i) $E>0$ and (ii) $0<E<V_0$, illustrating the nature of the solutions with rough sketches. 2+2

(c) Calculate the transmission coefficient for the latter case and compare it with the classical result. Show that the sum of transmission and reflection coefficient is unity. 2+1+1

8. Consider the Gaussian wave packet $e^{-x^2/2\sigma^2}.e^{ipx/\hbar^x}$. 3+7

(a) Normalise the function.

(b) Show that it corresponds to minimum uncertainty product.

9. (a) Write down Schrödinger equation for the electron of H-atom assuming the nucleus to be stationary. By separation of variables, obtain the radial equation. The normalised wavefunction of the ground state of H-atom is given by 2+3+3+2

$$\psi(r)=\frac{1}{\sqrt{\pi}\,a_0^{3/2}}\,e^{-r/a_0}$$

where a_0 = Bohr radius. Find the distance from the nucleus at which the electron is most likely to be found.

(b) What do you mean by degeneracy of a state?

10.(a) At time $t=0$, the wavefunction of hydrogen atom is (2+1)+4+3

$$\psi(\vec{r},\,0)=\frac{1}{\sqrt{10}}(2\psi_{100}+\psi_{210}+\sqrt{2}\,\psi_{211}+\sqrt{3}\,\psi_{2,\,1,\,-1})$$

the subscripts indicate the quantum numbers *n*, *l* and *m*. Find

(i) the expectation value of energy of the system

(ii) the probability of finding the system with $l=1,\ m=1$?

(b) Discuss the quantum mechanical theory of anomalous Zeeman effect, with special reference to Zeeman pattern for D_1 and D_2 lines of sodium. Draw a neat diagram to illustrate the Zeeman splitting of D_1 and D_2 lines of sodium.

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