'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2022

CC1-MATHEMATICS

CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Find $\int \sin^4 x \cos^2 x \, dx$.

(b) Find the points of inflexion, if any, of the curve $x = (\log y)^3$.

(c) Obtain reduction formula for $\int \tan^n x \, dx$, n being a positive integer, greater than 1.

(d) Obtain the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$; $2x + 3y + 4z = 8$ is a great circle.

(e) Find the solution of the differential equation $(x^2 + y^2 + 2x)\,dx + 2y\,dy = 0$.

(f) Find the differential equation of all circles, which pass through the origin and whose centres are on the x-axis.

GROUP-B

2. Answer any ***four*** questions from the following: 6×4 = 24

(a) (i) State Leibnitz's theorem on successive derivatives. 2

(ii) If $y = (\sin^{-1} x)^2$, prove that 4

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0 .$$

(b) Find the volume of the solid generates by revolving the cardioid $r = a(1-\cos\theta)$, about the initial line. 6

(c) If 'g' is a variable tangent of the conic $\frac{l}{r} = 1 - e\cos\theta$, show that the locus of the foot of the perpendicular from the pole on 'g' is the circle $r^2(e^2-1) + 2elr\cos\theta + l^2 = 0$. 6

(d) (i) Show that $\frac{1}{x^2}$ is an integrating factor of $xdy - ydx = 0$. 3

(ii) Solve the differential equation 3

$$(xy^2 - e^{1/x^3})dx - x^2 y\, dy = 0 .$$

(e) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx$, show that $2(n-1)I_n = 1+(n-2)I_{n-1}$ and hence deduce that $I_n = \frac{1}{n-1}$. 6

(f) Solve the differential equation 3+3

(i) $\left(x + y\cos\frac{y}{x}\right)dx = x\cos\frac{y}{x}\,dy$

(ii) $\frac{dy}{dx} = \sin(x+y)$

GROUP-C

3. Answer any ***two*** questions from the following: 12×2 = 24

(a) (i) Find the rectilinear asymptotes of the curve 6+6

$$x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 = 2.$$

(ii) If $I_n = \int_0^1 x^n \tan^{-1} x \, dx$, then show that

$$(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$$

(b) (i) Find the values of α, β such that 4+4+4

$$\lim_{x\to 0} \frac{\alpha \sin 2x - \beta \sin x}{x^3} = 1$$

(ii) Find $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$

(iii) If $y = \sin ax$, a is constant, then show that

$$\frac{d^n y}{dx^n} = a^n \sin\left(ax + \frac{n\pi}{2}\right),\ n \in \mathbb{N}.$$

(c) (i) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ about its base. 6+6

(ii) Solve the differential equation

$$\frac{dy}{dx} + \frac{1}{(1+x^2)}y = \frac{e^{\tan^{-1}x}}{1+x^2}$$

(d) Solve the differential equation 6+6

(i) $x^3\frac{dy}{dx} = y^3 + y^2\sqrt{(y^2 - x^2)}$

(ii) $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$.

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