'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2022

CC1-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: $1\times5 = 5$

(a) Evaluate $\lim\limits_{\theta\to 0}\dfrac{\tan\theta}{\theta}$

(b) Find out whether $\sin(\omega t)$ and $\cos(\omega t)$ can be two solutions of a second order homogeneous ordinary differential equation.

(c) If $\vec{A}$ and $\vec{B}$ are two irrotational vectors then prove that $\vec{A}\times\vec{B}$ is solenoidal.

(d) Evaluate $\int_0^4 e^{-3t}\delta(t-3)\,dt$.

(e) State Stokes' theorem in vector analysis.

(f) What is the expression of unit vector perpendicular to both $\vec{A}$ and $\vec{B}$?

(g) Calculate the probability of obtaining 4 heads in 6 tosses using an unbiased coin.

(h) Find the order and degree of the following equation: $\left(\dfrac{dy}{dx}\right)^4 + 3x\dfrac{d^2y}{dx^2} = 0$

GROUP-B

Answer any *three* questions from the following $5\times3 = 15$

2. (a) Find the Maclaurin's series for the integral $\int\dfrac{\sin x}{x}\,dx$. 2

(b) Solve differential equation $2xy\dfrac{dy}{dx} = x^2 + y^2$. 3

3. (a) Prove that $\vec{\nabla}\cdot(r^3\,\vec{r}) = 6r^3$. $2\frac{1}{2}$

(b) Find the value of 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2). $2\frac{1}{2}$

4. Express $\nabla^2\phi$ in cylindrical co-ordinate system, where ϕ is a scalar function. 5

5. (a) Prove that $\delta(kx)=\frac{1}{|k|}\delta(x)$, $k>0$. $2\frac{1}{2}$

(b) Evaluate $\int_{-1}^{+1} 9x^3\delta(3x+1)\,dx$ $2\frac{1}{2}$

6. (a) Show that $\vec{\nabla}f(r)=\frac{df}{dr}\hat{r}$, where $\hat{r}$ is the unit vector along vector $\vec{r}$. 3

(b) Obtain an expression for variance of Poisson Distribution. 2

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. (a) Determine whether $f'(0)$ exists using the definition of derivative for $f(x)=|x|$. 3+3+4

(b) Find the equation of the tangent line to $x^2+y^2=9$ at the point $(2, \sqrt{5})$.

(c) Three variables p, V and T are connected by the relation $f(p, V, T)=0$.

Show that $\left(\frac{\partial V}{\partial p}\right)\left(\frac{\partial p}{\partial T}\right)\left(\frac{\partial T}{\partial V}\right)=-1$

8. (a) If $\vec{r}(t)$ be a vector of fixed magnitude, show that $\frac{d\vec{r}(t)}{dt}$ is perpendicular to $\vec{r}(t)$. 3+5+2

(b) Express curl of a vector in orthogonal curvilinear co-ordinates and hence deduce it in spherical polar co-ordinates.

(c) Write down the scale factor for cylindrical polar co-ordinates.

9. (a) Find the integrating factor of the differential equation (2+2)+6

$$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1.$$

Hence find the general solution.

(b) Evaluate $\iint_s \vec{A}\cdot\hat{n}\,ds$ where, $\vec{A}=z\hat{i}-x\hat{j}-3y^2z\hat{k}$ and s is the surface of the cylinder $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$.

10.(a) Show that the Dirac-Delta function can be represented as limiting case of the Gaussian function. 4+2+4

(b) If A and B are events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{1}{2}$ then find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$.

(c) Using Lagrange's multiplier method, show that the rectangle of maximum area that can be inscribed in a circle is a square.

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