



‘समानो मन्त्रः समितिः समानी’

**UNIVERSITY OF NORTH BENGAL**  
B.Sc. Honours 3rd Semester Examination, 2022

**CC5-PHYSICS**

**MATHEMATICAL PHYSICS-II**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.*

**GROUP-A**

1. Answer any **five** questions from the following: 1×5 = 5
- (a) If the Fourier series expansion of a function  $f(x)$  contains only sine terms, what kind of function is  $f(x)$  ?
- (b) What are the Dirichlet's conditions?
- (c) What do you mean by cyclic co-ordinates?
- (d) What is the orthogonality condition of Bessel's polynomials?
- (e) Evaluate  $\int_{-1}^{+1} [P_2(x)]^2 dx$ .
- (f) Evaluate  $\Gamma(1)$ .
- (g) What is the singular point of a second order ordinary differential equation?
- (h) Define the error function.

**GROUP-B**

Answer any **three** questions from the following

5×3 = 15

2. Find Fourier series for the function 4+1

$$f(t) = \begin{cases} \frac{\pi - t}{2}, & 0 < t < 2\pi \\ 0, & t = 0, 2\pi \end{cases}$$

Integrate the obtained series term-by-term.

3. (a) Evaluate the integral  $\int_0^1 \sqrt{\ln\left(\frac{1}{x}\right)} dx$  3

- (b) Show that,  $\int_0^{x^2} \frac{e^{-t}}{\sqrt{t}} dt = \sqrt{\pi} \operatorname{erf}(x)$  2

4. (a) Show that  $xP'_n(x) = nP_n(x) + P'_{n-1}(x)$  3  
 (b) Show that  $P_n(-x) = (-1)^n P_n(x)$  2
5. Show that the extremal of the functional 5  

$$\int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{x} dx$$
 is a circle with its centre on the  $y$ -axis.
6. From the product of the generating functions  $g(x, t) \cdot g(-x, t)$  show that 5  

$$J_0^2 + 2J_1^2 + 2J_2^2 + \dots = 1$$
 where  $J_n(x)$  are the Bessel functions.

**GROUP-C**

**Answer any two questions from the following**

10×2 = 20

7. (a) What do you mean by non-periodic function? Give an example. 2  
 (b) How can a non-periodic function be expanded in Fourier series? 2  
 (c) Find the Fourier series for the function 4+2  

$$f(x) = |x|, -l < x < l.$$
 Hence show that  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \pi^2/8$
8. (a) Solve the following ODE using power series method. 7  

$$y'' - 2xy' + 2xy = 0$$
  
 (b) Express the function  $f(x) = |x|$  in the interval  $-1 \leq x \leq 1$  in terms of Legendre polynomial. 3
9. (a) Solve the following partial differential equation using separation of variables method. 5  

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$$
  
 (b) Starting from the generating function prove the recurrence relation 5  

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$$
- 10.(a) State Hamilton's principle and explain. 2  
 (b) Show using variational principle that the shortest distance between two points in a plane is a straight line. 4  
 (c) Show that translational symmetry in a system leads to conservation of linear momentum. 4

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