'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2022

CC12-MATHEMATICS

NUMERICAL METHODS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: 1×5 = 5

   (a) Evaluate $(\Delta^2/E)\,(x^3)$.

   (b) Evaluate $\Delta^3(2x^2 - 2x + 1)$, Δ has its usual meaning.

   (c) Discuss the convergency of the fixed point iteration method.

   (d) Find the Lagrange's interpolation polynomial fitting the data points $f(1) = 6$, $f(3) = 0$, $f(4) = 12$ for some function $f(x)$.

   (e) When does Newton-Raphson method fail to find a real root of the equation $f(x) = 0$?

   (f) Find the number of significant figure in $V_T = 1.5923$ given its relative error as 0.1×10^{-3}.

   (g) Calculate the absolute, relative and percentage errors by approximating 5/3 with 1.6667.

   (h) State the condition of convergence of Gauss-Seidel iteration method for solving numerically a system of linear equations.

GROUP-B

2. Answer any ***three*** questions from the following: 5×3 = 15

   (a) Construct an appropriate difference table in respect of a function $f(x)$ using the following data table and find $f(0.5)$. 5

x:	0	1	2	3
y:	1	2	11	34

   (b) Use the method of separation of symbols to prove: 5

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \cdots$$

$$= e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots\right)$$

   Prove that $\Delta \cdot \nabla = \Delta - \nabla$.

(c) Use modified Euler method to compute $y(0.4)$ for the following initial value problem 5

$$\frac{dy}{dx} = x + y,\ y(0) = 1$$

taking $h = 0.1$.

(d) Show that the order of convergence of Secant method is approximately 1.618. 5

(e) Compute $\sqrt{2}$ to four significant figures using Newton-Raphson method. 5

GROUP-C

3. Answer any ***two*** questions from the following: 10×2 = 20

(a) (i) Evaluate by Trapezoidal rule taking $h = 1$. 5+5

$$\int_0^5 \frac{dx}{(1+x)}$$

(ii) Evaluate by Simpson's $\frac{1}{3}$-rule taking six intervals correct up to 4 significant figures:

$$\int_0^{\pi/2} \sqrt{\sin x}\ dx$$

(b) (i) Write down the iteration scheme of Secant method and find the condition of convergence of Secant method. 5+5

(ii) Use Picard's method for solving the differential equation

$$\frac{dy}{dx} = x^2 + y^2 \quad \text{for } x = 0.4$$

given $y = 0$ when $x = 0$.

(c) (i) Find the missing term in the following table: 3+3+4

x	1	2	3	4	5
$f(x)$	–2	3	8	—	21

(ii) Find the degree of precision of the quadrature formula

$$\int_{-1}^{1} f(x)\,dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

(iii) Construct the second degree least square approximation to $f(x) = \cos(\pi x)$ over $[-1, 1]$.

(d) (i) What is the main difference of Secant and Regula-Falsi method? Use Secant method to find a real root of the equation $x^3 - 9x + 1 = 0$ correct up to 3 decimal places. 5+5

(ii) Use Runge-Kutta method of order two to compute $y(0.4)$ from $\frac{dy}{dx} = x + y$ with $y(0) = 1$ taking $h = 0.1$.

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