'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2022

DSE-P1-MATHEMATICS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains DSE1A and DSE1B. Candidates are required to answer any *one* from the *two* DSE1 courses and they should mention it clearly on the Answer Book.

DSE1A

PROBABILITY AND STATISTICS

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) If A_n is an increasing sequence of events $(A_n \subseteq A_{n+1})$, show that

$$\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)\ .$$

(b) The joint pdf of the random variable X and Y are given by

$$f(x,y) = \begin{cases} \frac{1}{8}(x+y)\,, & 0 \le x \le 2,\quad 0 \le y \le 2 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Check whether X and Y are independent.

(c) Let X be a random variable with following probability distribution:

x	-3	6	9
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$ and using the law of expectation, evaluate $E[(2X+1)^2]$.

(d) An integer is chosen at random from two hundred digits. What is the probability that the integer is divisible by 5 or 6?

(e) If T be an unbiased estimator of θ, then show that $\sqrt{T}$ is a biased estimator of $\sqrt{\theta}$.

(f) If $\rho(X,Y)$ is the correlation coefficient between two random variables X and Y, prove that $-1 \le \rho \le 1$.

GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) If X is a normal (m,σ) variate, prove that $P(a<X<b)=\Phi\left(\frac{b-m}{\sigma}\right)-\Phi\left(\frac{a-m}{\sigma}\right)$ and $P(|X-m|>a\sigma)=2[1-\Phi(a)]$, where $\Phi(x)$ denotes the standard normal distribution function.

(b) For the Binomial (n, p) distribution, prove that

$\mu_{k+1}=p(1-p)\left(nk\mu_{k-1}+\frac{d\mu_k}{dp}\right)$ and hence calculate γ_1 and γ_2.

(c) Suppose $y=g(x)$ be a continuously differentiable function which is strictly monotonic everywhere. If $f_x(x)$ be the density function of the random variable X, then show that the density function of $Y=g(X)$ is $f_y(y)=f_x(x)\left|\frac{dx}{dy}\right|$.

(d) A rod of length a is broken into three parts at random. Find the probability of their forming a triangle.

(e) The joint pdf of the random variables X and Y are given by

$$f(x,y)=\begin{cases} k(3x+y), & 1\le x\le 3,\quad 0\le y\le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the following:

(i) The value of k,

(ii) $P(X+Y<2)$,

(iii) Check whether X and Y are independent.

(f) Suppose X and Y be two jointly distributed random variables. Show that

$[E(XY)]^2\le[E(X)]^2\,[E(Y)]^2$.

Hence deduce that

$-1\le\rho(X,Y)\le 1$,

where $\rho(X,Y)$ is the correlation coefficient between X and Y.

GROUP-C

3. Answer any ***two*** questions: 12×2 = 24

(a) (i) Suppose X and Y be two independent random variable each having the same probability density function $f(x)=2xe^{-x^2}$, $x>0$. 6

Find the pdf of $\sqrt{X^2+Y^2}$.

(ii) Suppose $X_1, X_2, \cdots, X_n$ be a random sample of size n taken from a $N(0, \sigma)$ distribution. Show that $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ is an unbiased estimator of σ^2. 6

(b) (i) If X is a $\Gamma(l)$ variate and Y is an independent $\Gamma(m)$ variate, then show that $\frac{X}{X+Y}$ is $\beta_1(l, m)$ variate. 6

(ii) The height of 10 males of normal population are found to be 70, 67, 62, 67, 61, 68, 70, 64, 65, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Given that $P(\chi^2 > 16.92) = 0.05$, $P(t > 1.83) = 0.05$ for 9 degrees of freedom. 6

(c) (i) Find the characteristic function of the distribution defined by the probability density function $f(x)$ given by $f(x) = \frac{1}{2\lambda} e^{\frac{-|x-\mu|}{\lambda}}$, $\lambda > 0$, $-\infty < x < \infty$. 6

(ii) A point P is chosen at random on a line segment AB of length $2a$. Find the probability that the value of $AP \cdot PB$ will exceed $\frac{a^2}{2}$. 6

(d) (i) A sample is drawn from a $N(m, \sigma)$ population. Design a test for $H_0 : m = m_0$ against the alternative hypothesis $H_1 : m < m_0$ assuming the population variance to be known. 6

(ii) 171 out of 300 votes picked at random from a large electorate and it is said that they were going to vote for a particular candidate. Find 95% confidence interval for the population of voters of the electorate, who would vote in favour of the candidate. 6

DSE1B

LINEAR PROGRAMMING

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) Solve the LPP: Maximize $Z = 4X_1 + 7X_2$

Subject to: $12X_1 + 7X_2 \le 42$;

$5X_1 + 4X_2 \le 20$, $2X_1 + 3X_2 \ge 6$,

$X_1, X_2 \ge 0$ graphically.

(b) Examine whether the set

$$X = \{(X_1, X_2) \in E^2 : 2X_1 + X_2 \ge 20, X_1 + 2X_2 \le 80,\ X_1 + X_2 \le 50;\ X_1, X_2 \ge 0\}$$

is a convex set.

(c) Prove or disprove: The set of all feasible solutions of an LPP is a convex set.

(d) Find the dual of the following LPP:

Maximize: $Z = 3X_1 + 2X_2$

Subject to: $3X_1 + 4X_2 \leq 22$;

$3X_1 + 2X_2 \leq 16$

(e) Prove that the following payoff matrix has no saddle point:

$$\begin{array}{cc} & \begin{array}{ccc} & B & \\ I & II & III \end{array} \\ A \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} 4 & 5 & 2 \\ 1 & 4 & 6 \\ 3 & 1 & 6 \end{bmatrix} \end{array}$$

(f) Find all the basic solutions of the system of equations:

$2X_1 + X_2 + 4X_3 = 11$

$3X_1 + X_2 + 5X_3 = 14$

GROUP-B

2. Answer any ***four*** questions: 6×4 = 24

(a) Solve the following 2×3 game graphically:

	Player B		
Player A	1	3	11
	8	5	2

(b) Solve the following problem by two-phase method:

Maximize $Z = 5X_1 + 3X_2$

Subject to: $2X_1 + X_2 \leq 1$

$3X_1 + 4X_2 \geq 16$

and $X_1,\ X_2 \geq 0$

(c) Formulate the dual of the following LPP: 6

Maximize $Z = 2X_1 + 3X_2 + 4X_3$

Subject to $X_1 - 5X_2 + 3X_3 = 7$

$2X_1 - 5X_2 \leq 3$

$3X_1 - X_3 \geq 5$

$X_1,\ X_2 \geq 0$ and X_3 is unrestricted in sign.

(d) Solve the following 4×4 game and prove that there are two saddle points:

	B_1	B_2	B_3	B_4
A_1	4	2	3	5
A_2	–2	–1	4	–3
A_3	5	2	3	3
A_4	4	0	0	1

(e) Define basic solution and basic feasible solution. Find the solution of the system of equations and test whether it is basic or not

$$X_1 + X_2 + X_3 = 2$$

$$X_1 + X_2 - 3X_3 = 2$$

$$2X_1 + 4X_2 + 3X_3 - X_4 = 4$$

and $X_1,\ X_2,\ X_3,\ X_4 \geq 0$

(f) Find the optimal assignment and minimum cost for the assignment problem with following cost matrix: 6

	M_1	M_2	M_3	M_4
J_1	10	12	9	11
J_2	5	10	7	8
J_3	12	14	13	11
J_4	8	15	11	9

GROUP-C

3. Answer any ***two*** questions: 12×2 = 24

(a) (i) Solve by two-phase method: 7

Max $Z = 2X_1 + X_2 - X_3$

Subject to: $4X_1 + 6X_2 + 3X_3 \leq 8$

$$3X_1 - 6X_2 - 4X_3 \leq 1$$

$$2X_1 + 3X_2 - 5X_3 \geq 4$$

$$X_1,\ X_2,\ X_3 \geq 0$$

(ii) Prove that the objective function of a LPP assumes its optimal value at an extreme point of the convex set of feasible solution. 5

(b) (i) In a rectangular game the pay-off matrix is given by 6

	B_1	B_2	B_3	B_4	B_5
A_1	10	5	5	20	4
A_2	11	15	10	17	25
A_3	7	12	8	9	8
A_4	5	13	9	10	5

State whether the player will use pure or mixed strategies. What is the value of the game?

(ii) For the following Pay-off table, transform the game into an equivalent LPP and solve it by Simplex method. 6

$$\begin{array}{c@{\quad}c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \end{array}$$

(c) (i) Solve the LPP 6

Maximize $Z = 3X_1 + X_2 + 3X_3$

Subject to: $2X_1 + X_2 + X_3 \le 2$

$X_1 + 2X_2 + 3X_3 \le 5$

$2X_1 + 2X_2 + X_3 \le 6$

$X_1,\ X_2,\ X_3 \ge 0$

(ii) Find the optimal solution and the corresponding cost of transportation in the following transportation problem: 6

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

(d) Use duality to solve the following problem: 12

Maximize $Z = 3X_1 - 2X_2$

Subject to: $X_1 \le 4,\ X_2 \le 6,\ X_1 + X_2 \le 5,\ X_2 \ge 1$

and $X_1,\ X_2 \ge 0$

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