'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2022

CC11-PHYSICS

QUANTUM MECHANICS AND APPLICATIONS

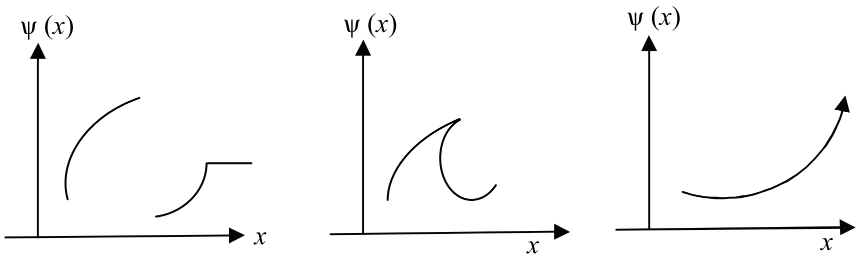
Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions: 1×5 = 5

(a) Which of the following plots represent a valid wave-function for the Schrödinger's equation? Justify your answer.



(b) When we measure the energy of a quantum particle in a box, is it possible that the measurement may result in a smaller value than the ground state energy? If so, why?

(c) If a classical harmonic oscillator can be at rest, why can the quantum harmonic oscillator never be at rest? Does this violate Bohr's correspondence principle?

(d) When an electron and a proton of same kinetic energy encounter a potential barrier of the same height and width, which one of them will tunnel through the barrier more easily and why?

(e) What is meant by degenerate and non-degenerate state? What is the origin of degeneracy?

(f) What is the origin of fine-structure of a H-atom?

(g) Write down the wave-function of a particle localized at the point $x = 0$. What will be the corresponding momentum space wave-function?

(h) Plot the effective potential,

$$V_{eff}(r) = -\frac{Ze^2}{r} + \frac{l(l+1)\hbar^2}{2\mu r^2}$$

against r for $l = 0$, 1 and 2

GROUP-B

Answer any *three* questions 5×3 = 15

2. An electron is described by the following wave-function 2+3

$$\psi(x) = 0 \text{ for } x < 0, \text{ and } \psi(x) = Ce^{-x}(1 - e^{-x}) \text{ for } x > 0,$$

where C is a constant and x is the position co-ordinate.

(i) Where is the electron most likely to be found?

(ii) Calculate the average position $\langle x \rangle$ for the electron. Compare this result with the most likely position, and comment on the difference.

3. Let $\psi_1(x_1)$ and $\psi_2(x_2)$ be two normalized wave-functions of two identical particles. Construct the properly normalized wave-functions if the particles are following (i) Maxwell-Boltzmann distribution, (ii) Bose-Einstein distribution and (iii) Fermi-Dirac distribution. Show that your result of (iii) is consistent with Pauli's exclusion principle. 3+2

4. Consider a stream of particles of mass 'm', each moving in the positive x-direction with kinetic energy E towards the potential barrier given by, 5

$$V(x) = 0 \text{ for } x \le 0$$

$$V(x) = \frac{3}{4}E \text{ for } x > 0$$

Find out the fraction of particles reflected at $x = 0$

5. Using the quantum particle in a 1-D box model, determine how the possible energies of the particle are related to the length of the box. Are the energy eigenstates also eigenstates of linear momentum? If not, why? 3+2

6. If a dynamical variable α is represented quantum-mechanically by the operator $\hat{\alpha}$ which does not explicitly depend on time, show that 4+1

$$\frac{d}{dt}\langle\hat{\alpha}\rangle = \frac{i}{\hbar}\langle\hat{H}\hat{\alpha} - \hat{\alpha}\hat{H}\rangle,$$

where $\hat{H}$ is the Hamiltonian operator. Hence find out the condition when a dynamical variable would be a constant of motion.

GROUP-C

Answer any *two* questions 10×2 =20

7. (a) The Hamiltonian of a linear harmonic oscillator is given by $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ ($k > 0$ is a constant). Explain what will be the influence of an extra potential $V(x) \propto x$ on the system. 2

(b) Draw the linear harmonic oscillator potential against x and also the eigen functions $\psi_n(x)$ within the potential for n = 0, 1, 2, 3. 3

(c) Find out the second excited state of the harmonic oscillator whose ground-state wavefunction is given by, 5

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

8. (a) What do you understand by a complete set of commuting observables? What is its significance? 2

(b) Evaluate the following commutators: 4

$$[L_z,\ r^2] \text{ and } [L_z,\ p^2]$$

(c) The normalized ground state wavefunction of a hydrogen atom is given by 4

$$\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} \exp\left(-\frac{r}{a}\right)$$

where 'a' is the Bohr radius and r is the distance of the electron from the nucleus located at the origin. Show that the expectation value $\left\langle \frac{1}{r^2} \right\rangle$ is $\frac{2}{a}$.

9. (a) Why silver atoms were used in the Stern-Gerlach experiment? 1

(b) What is predicted to happen for electron beams in the Stern-Gerlach experiment? 1

(c) In the Stern-Gerlach experiment, why there is a non-zero force even though the atoms were electrically neutral? 2

(d) How does the Stern-Gerlach experiment lead to the conclusion of electron spin? 2

(e) What will happen if the splitted beams of silver atoms are passed through a non-uniform magnetic field in the X-direction? 2

(f) What would happen to the result of Stern-Gerlach experiment if a beam of spin-1 particles are used? 2

10.(a) A particle of mass m in a 1-D box of width ℓ is allowed to be in the ground state (ψ_1) and also in the 1st excited state (ψ_2). The composite wavefunction is given by 3

$$\psi(x) = \frac{1}{\sqrt{3}}\,\psi_1(x) + \sqrt{\frac{2}{3}}\,\psi_2(x)$$

What will be the average value of its energy?

(b) Let ψ_{nlm} represent the wavefunction of a H-atom. Let 3

$$\psi = \left[\frac{1}{\sqrt{14}}\psi_{211} - \frac{2}{\sqrt{14}}\psi_{210} + \frac{3}{\sqrt{14}}\psi_{21,-1}\right]$$

Is ψ an eigenfunction of L^2 and L_z? Justify your result.

(c) If the ground state wave-function of a linear harmonic oscillator is given by, 4

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right)$$

find out the corresponding wave-function in the momentum space.

——×——