



'সমানো মন্ত্র: সমিতি: সমানী'

UNIVERSITY OF NORTH BENGAL
B.Sc. Honours 4th Semester Examination, 2022

GE2-P2-STATISTICS

FUNDAMENTAL OF PROBABILITY THEORY

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.
All symbols are of usual significance.*

GROUP-A

1. Answer any **five** questions:

1×5 = 5

- (a) Show that the chance of throwing an odd number with a die is $\frac{1}{2}$.
- (b) What is the chance that a leap year selected at random will contain 53 Sundays?
- (c) The probability density function of a random variable X is

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0, \quad \theta > 0$$
$$= 0, \quad \text{otherwise}$$

Find $E(X^2)$.

- (d) A speaks truth in 75% and B in 80% of the cases. In what percentage of cases, are they likely to contradict each other in stating the same fact?
- (e) Give chief features of the normal curve.
- (f) The mean and the variance of X are 10 and 4 respectively. Find the mean and variance of $5 - 2X$.
- (g) Distinguish between p.m.f. and p.d.f.
- (h) Find the median of a binomial distribution for $n = 9$, $p = \frac{1}{2}$.

GROUP-B

2. Answer any **three** questions:

5×3 = 15

- (a) Derive Poisson distribution as the limit of binomial distribution.
- (b) State and prove Bayes' theorem.

- (c) Let X be a binomially distributed random variable with parameters n and p . For what value of p is $\text{var}(X)$ a maximum, if you assume that n is fixed?
- (d) If X has Poisson distribution with parameter λ , then show that

$$P[X \text{ is even}] = \frac{1}{2}[1 + e^{-2\lambda}]$$

- (e) Find the points of inflection of the normal curve.

GROUP-C

3. Answer any *two* questions: 10×2 = 20

- (a) Find the mean and variance of normal distribution. 5+5
- (b) (i) Show that the expectation of the sum of two jointly distributed random variables X and Y is the sum of their expectations. 5+5
- (ii) Prove the recurrence relation between the moments of Poisson distribution

$$\mu_{r+1} = \lambda \left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda} \right)$$

where $\mu_r = \sum_{x=0}^{\infty} (x - \lambda)^r \frac{e^{-\lambda} \lambda^x}{x!}$ is the r -th moment about the mean λ .

- (c) (i) Show that odd order central moments of the normal distribution are equal to zero. 5+5
- (ii) The joint p.d.f. of (X, Y) is given by

$$f(x, y) = 2 \quad ; \quad 0 < x < 1, \quad 0 < y < x$$

$$= 0 \quad ; \quad \text{otherwise}$$

Find the marginal density of X and the conditional density of Y given $X = x$.

- (d) (i) Let the variable X have the distribution $P(X = 0) = P(X = 2) = p$, $P(X = 1) = 1 - 2p$, for $0 \leq p \leq \frac{1}{2}$. For what value of p is the $\text{var}(X)$ maximum? 5+5
- (ii) Find the expected value of the product of points obtained on rolling n dice together.

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