



'সমানো মন্ত্র: সমিতি: সমানী'

UNIVERSITY OF NORTH BENGAL
B.Sc. Programme 4th Semester Examination, 2022

DSC1/2/3-P4-STATISTICS

METHODS OF STATISTICAL INFERENCE

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.
All symbols are of usual significance.*

GROUP-A

1. Answer any **five** questions from the following: 1×5 = 5
- (a) What is estimation of a parameter?
 - (b) What is power of a test?
 - (c) What are the desirable criteria of a good point estimator?
 - (d) Define an unbiased estimator.
 - (e) What is level of significance?
 - (f) What do you mean by efficiency of an estimator?
 - (g) What is testing of hypothesis?
 - (h) What are the sufficient conditions for consistency?

GROUP-B

2. Answer any **three** questions from the following: 5×3 = 15
- (a) What is method of moments? Consider the pdf $f_{\theta}(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$; $x > 0$. Estimate θ by method of moments.
 - (b) What is likelihood function? What are the properties of maximum likelihood estimation?
 - (c) If x_1, x_2, \dots, x_n are random observations on a Bernoulli variate x taking the value 1 with probability p and the value 0 with probability $(1-p)$, show that $\bar{x}(1-\bar{x})$ is a consistent estimator of $p(1-p)$.
 - (d) On the basis of a random sample, find the mle (maximum likelihood estimation) of the parameter of the Binomial distribution.

- (e) If the variances of independent and unbiased estimators T_1, T_2, T_3 of θ are in the ratio 2 : 3 : 5, which of the following estimators of θ would you prefer?

$$\frac{2T_1 + T_2 + T_3}{4}, \frac{T_1 + 2T_2 + T_3}{4}, \frac{T_1 + T_2 + 2T_3}{4}$$

GROUP-C

3. Answer any *two* questions from the following: 10×2 = 20

- (a) (i) Let x_1, x_2, \dots, x_n be a random sample from $N(\mu, \sigma^2)$. Find the interval estimation of μ when σ^2 known. 5+5

- (ii) Prove that sample variance $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ is an unbiased estimator of the population variance σ^2 .

- (b) (i) What is uniformly minimum variance unbiased estimator? 2+2+6

- (ii) Define Cramer-Rao Inequality.

- (iii) Suppose x_1, x_2, \dots, x_n are random sample from $N(\mu, \sigma^2)$, the parameter σ^2 being known. The unknown parameter is $\theta = \mu$. Prove that \bar{x} is a minimum variance bound estimator for θ .

- (c) (i) A simple random sample (x_1, x_2, x_3, x_4) of size 4 is drawn from an infinite population with mean μ and sd (standard deviation) σ . Given the two estimators of μ as follows: 5+5

$$T_1 = \frac{x_1 + 2x_2 + 3x_3 + 4x_4}{10} \quad \text{and} \quad T_2 = \frac{x_1 + x_2}{3} + \frac{x_3 + x_4}{6}$$

Which one is better? Why?

- (ii) Let x_1, x_2, \dots, x_{n_1} and y_1, y_2, \dots, y_{n_2} be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Find the $100(1-\sigma)\%$ confidence interval for $(\mu_1 - \mu_2)$ when both σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

- (d) What is large sample test? In a big city, 325 men out of 600 were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers? 3+7

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