'समानो मन्त्रः समितिः समानी'

# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2022

## CC14-MATHEMATICS

### PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 60

*The figures in the margin indicate full marks.*
*All symbols are of usual significance.*

### GROUP-A

**Answer any *four* questions from the following** 3×4 = 12

1. Solve the partial differential equation $p\tan x + q\tan y = \tan z$. 3

2. Eliminate the arbitrary function $f$ to obtain a partial differential equation from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. 3

3. Find the region in the $xy$-plane where the partial differential equation $\{(x-y)^2 - 1\}z_{xx} + 2z_{xy} + \{(x-y)^2 - 1\}z_{yy} = 0$ is hyperbolic. 3

4. Find the characteristic curves of $\sin^2 x\, z_{xx} + 2\cos x\, z_{xy} - z_{yy} = 0$. 3

5. Find the degree of the following PDE: 3

$$z_{xx}^2 + 2z_y + \sin(z_x) = x^2 y$$

Write down the relation between arbitrary constants, independent variables and order of a PDE.

6. Obtain a solution of the partial differential equation $xp + yq = z$ representing a surface passing through the parabola $y^2 = 4x$, $z = 1$. 3

### GROUP-B

**Answer any *four* questions from the following** 6×4 = 24

7. Apply $\sqrt{u} = v$ and $v(x, y) = f(x) + g(y)$ to solve the equation $x^4 u_x^2 + y^2 u_y^2 = 4u$. 6

8. Reduce the equation $y^2 u_{xx} - 2xy\, u_{xy} + y^2 u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ to a canonical form. 6

9. Solve the non-homogeneous wave equation: 6

$$\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} = \exp(-x), \quad -\infty < x < \infty, \quad t > 0$$

with the conditions: $u(x, 0) = x^2$, $u_t(x, 0) = \cos x$

10. A tightly stretched string of length $l$ with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $\sin^3(\pi x/l)$. Find the displacement of the string out any distance from one end at any time $t$. 6

11. Find the temperature distribution in a laterally insulated rod of length '$l$' whose ends are also insulated and the initial temperature is given by 6

$$u(x, 0) = \begin{cases} x & \text{if } 0 < x \le l/2 \\ l - x & \text{if } l/2 < x < l \end{cases}$$

Where $u(x, t)$ represents temperature distribution, $x$ is the spatial coordinate and $t$ is the time coordinate. Also $u(x, t)$ is bounded as $t \to \infty$.

12. Reduce the equation $u_{xy} + xu_{yy} = 0$, $x > 0$ to its canonical form. 6

## GROUP-C

**Answer any *two* questions from the following** 12×2 = 24

13.(a) Obtain the differential equation eliminating the arbitrary functions $f$ and $g$ from 6

$$z = y f(x) + x g(y)$$

(b) Reduce the equation $z_{xx} - 4z_{xy} + 4z_{yy} + z = 0$ to its canonical form. 6

14.(a) Obtain the general solution of wave equation for a semi-infinite string with free end boundary condition, given that initial deflection $u(x, 0) = f(x)$ and initial velocity $\frac{\partial u}{\partial t}(x, 0) = g(x)$ where $u(x, t)$ represents the vertical deflection of string, $x$ is the spatial coordinate and $t$ is the time coordinate. Also discuss the case when initial velocity is zero. 6

(b) Solve the PDE $\cos(x + y) z_x + \sin(x + y) z_y = z$. Classify the PDE. 5+1=6

15.(a) Show that the equations $xp - yq = x$, $x^2 p + q = xz$ are compatible and find their solution. 6

(b) Find the characteristic strips of the equation $xp + yq - pq = 0$ and obtain the equation of the integral surface through the curve $C: z = x/2$, $y = 0$. 6

16.(a) Use separation of variable to solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given $u = 3e^{-y} - e^{-5y}$ when $x = 0$. 4

(b) Use Lagrange's method to solve: $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ 4

(c) Use Charpit's method to solve: $p(p^2 + 1) + (b - z)q = 0$ 4

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