'समानो मन्त्रः समितिः समानी'

# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2022

## DSE-P3-MATHEMATICS

Time Allotted: 2 Hours | Full Marks: 60

*The figures in the margin indicate full marks.*
*All symbols are of usual significance.*

**The question paper contains DSE3A and DSE3B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.**

## DSE3A

## POINT SET TOPOLOGY

### GROUP-A

**Answer any *four* questions from the following** $3\times4 = 12$

1. Give an example of a continuous bijective map between two spaces which is not a homeomorphism. Justify your answer. 3

2. If $F(\mathbb{N})$ denotes the collection of all finite subsets of $\mathbb{N}$ then find cardinality of $F(\mathbb{N})$. 3

3. The co-countable topology on $\mathbb{R}$ is defined as the collection of all sets $U \subset \mathbb{R}$ so that $\mathbb{R}\backslash U$ is either countable or all of $\mathbb{R}$. Is [0, 1] a compact subspace of $\mathbb{R}$ with co-countable topology. 3

4. Show that $\overset{0}{\overline{\overset{0}{\overline{A}}}} = \overset{0}{\overline{A}}$ and $\overset{c}{\overline{A}} = \overset{0}{\overset{c}{A}}$, where $A^c$ means complement of $A$. 3

5. Let us consider $\mathbb{R}$ with cofinite topology. Find closure of $A$ and $B$ where $A$ is finite and $B$ is infinite. 3

6. Examine if every constant function $f:(X, J_1) \to (Y, J_2)$ is continuous. 3

### GROUP-B

**Answer any *four* questions from the following** $6\times4 = 24$

7. Prove that $2^a = c$, where $|\mathbb{N}| = a$ and $|\mathbb{R}| = c$. 6

8. Let $f:(X, J_X) \to (Y, J_Y)$ be a mapping then prove that the following are equivalent: 6

   (i) $f$ is continuous.

(ii) $f(\overline{A}) \subset \overline{f(A)}, \forall A \subset X$

(iii) for any closed set $C$ in $Y$, $f^{-1}(C)$ is closed in $X$.

9. Show that $\mathbb{R}$ with usual topology is not compact but $\mathbb{R}$ with cofinite topology is compact. 6

10. Let $X$ and $Y$ be connected spaces. Show that $X \times Y$ is connected. 6

11. Show that $\{(r, s);\ r < s,\ r, s \in Q\}$ is a basis for usual topology on $\mathbb{R}$ but $\{[r, s);\ r < s,\ r, s \in Q\}$ is not a basis for $\mathbb{R}_{\ell}$. 6

12.(a) Can we say that metric spaces are topological spaces? — Explain. 2

(b) Show that projection maps are continuous open but not closed. 4

## GROUP-C

**Answer any *two* questions from the following** 12×2 = 24

13.(a) Let $X$ be a compact Hausdorff space and let $(A_n)$ be a countable collection of closed sets in $X$. Show that if each set $A_n$ has empty interior in $X$, then the union $\bigcup_{n \in \mathbb{N}} A_n$ also has empty interior in $X$. 7

(b) Prove that continuous image of a connected space is connected. 5

14.(a) Show that the function $f : \mathbb{R}_{\ell} \longrightarrow \mathbb{R}$ defined by $f(x) = [x]\ \forall x \in \mathbb{R}$ is continuous. 6

(b) Show that $\mathbb{R}^n$ and $\mathbb{R}^m$ cannot be homeomorphism if $m \neq n$. 6

15.(a) By giving examples show that $a + c = c$, where $|\mathbb{N}| = a$ and $|\mathbb{R}| = c$. 5

(b) Show that closed subset of a compact space is compact but a compact subset of a topological space may not be closed. 5+2

16.(a) Show by an example that connectedness is necessary in the statement of intermediate value theorem. 3

(b) Let $A$ be a subset of a topological space $X$ and $(x_n)$ be a sequence in $A$ such that $x_n \to x$. Show that $x \in \overline{A}$. Also show that if $X$ is a metric space then the converse is true. 4+5

## DSE3B

## BOOLEAN ALGEBRA AND AUTOMATA THEORY

## GROUP-A

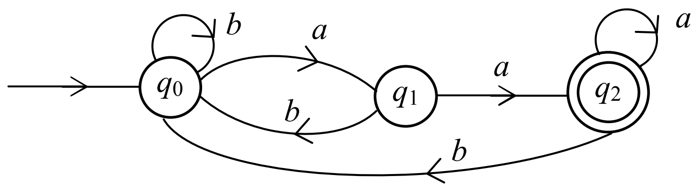
**Answer any *four* questions from the following** 3×4 = 12

1. Give an example of a bijective mapping between two ordered sets which is not an order isomorphism.

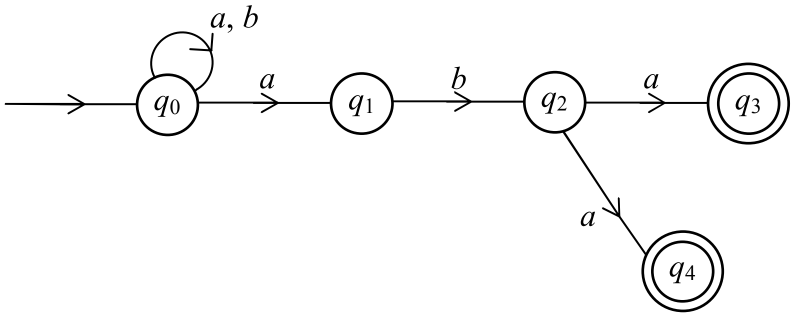
2. Show by an example that union of two sublattices of a lattice may not be a sublattice.

3. Reduce the Boolean term $((x_1 + x_2).(x_1' + x_3))'$ to DNF.

4. Identify the language $L(M)$ accepted by the automaton $M$ in the figure:




5. Let $M$ be the NFA whose state diagram is given below:




Write down the transition table for this NFA. Also find $L(M)$.

6. Let $\Sigma = \{0, 1\}$ and $T = \{\omega \in \Sigma^* : \omega \text{ contains even number of 1's}\}$. Show that $T$ is an accepted language.

## GROUP-B

**Answer any *four* questions from the following** 6×4 = 24

7. (a) Let $L$ and $K$ be two lattices and $f : L \to K$ be a map. Prove that $f$ is a lattice isomorphism iff it is order isomorphism. 4

   (b) Prove that every sublattice of a distributive lattice is also distributive. 2

8. (a) Let $L$ be a Boolean lattice. Then prove that for all $a, b \in L$, $a \wedge b' = 0$ iff $a \leq b$. 3

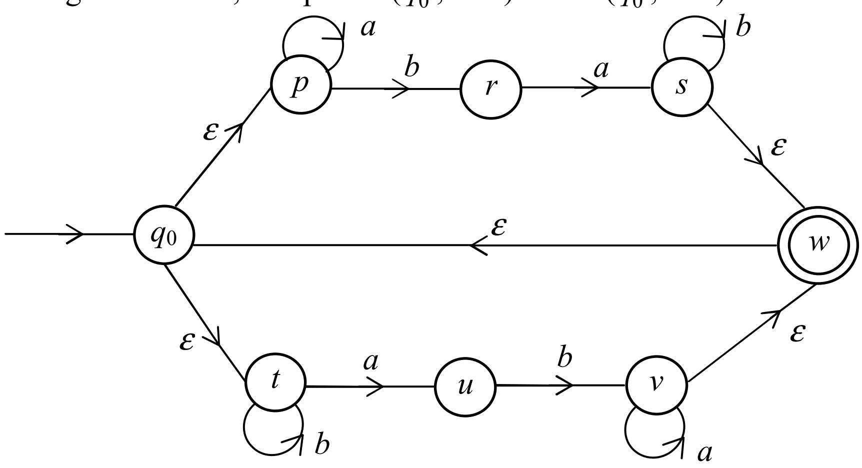
   (b) Let $X$ be any set. Define $FC(X) = \{A \subseteq X : A \text{ is finite OR } X \setminus A \text{ is finite}\}$. Prove that $(FC(X), \cup, \cap, ', \phi, X)$ is a Boolean Algebra. 3

9. Suppose a 4-variable Boolean term is given as follows: 6

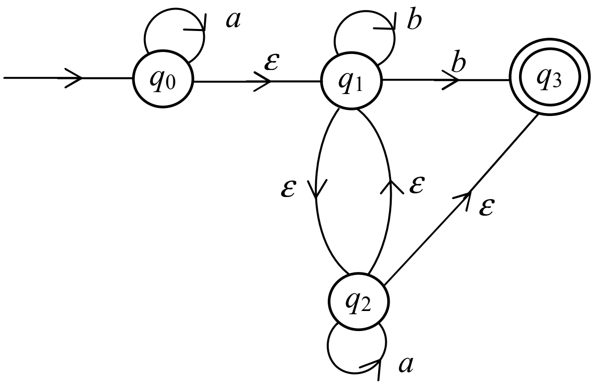
$$\phi = \sum m(0,\ 1,\ 2,\ 5,\ 7,\ 8,\ 9,\ 10,\ 13,\ 15)$$

Minimize $\phi$ using Karnaugh map.

10.(a) For the given $\varepsilon$-NFA, compute $\hat{\delta}(q_0, aba)$ and $\hat{\delta}(q_0, bba)$. 3




(b) Find epsilon closures of all the states of the given $\varepsilon$-NFA. 3




11. For $\Sigma = \{a, b, c\}$, design a Turing machine that accepts $L = \{a^n b^n c^n \mid n \geq 1\}$. 6

12.(a) Show that the language of palindromes over $\Sigma = \{a, b\}$ is a context free language. 4

(b) Distinguish between NFA and $\varepsilon$-NFA. 2

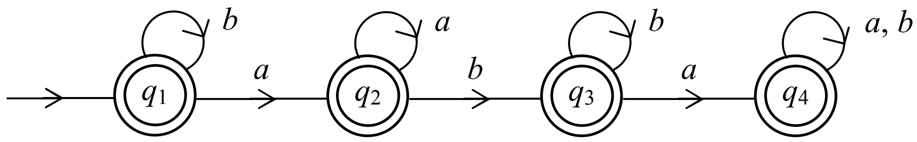
## GROUP-C

**Answer any *two* questions from the following** 12×2 = 24

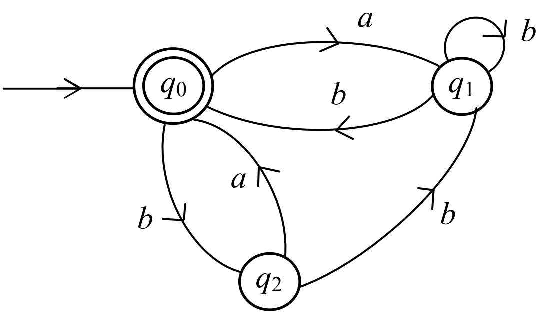
13.(a) Prove that a language $L$ is accepted by some DFA iff $L$ is accepted by some NFA. 6

(b) Find regular expression for the following DFAs: 3+3

(i)




(ii)



14.(a) Draw the transition graph of the NPDA, $M=(Q, \Sigma, \Gamma, \delta, q_0, z, F)$, where $Q=\{q_0, q_1, q_2\}$, $\Sigma=\{a, b\}$, $\Gamma=\{a, b, z\}$, $F=\{q_2\}$ and $\delta$ is given by: 6

$\delta(q_0, a, z)=\{(q_1, a), (q_2, \lambda)\}$

$\delta(q_1, b, a)=\{(q_1, b)\}$

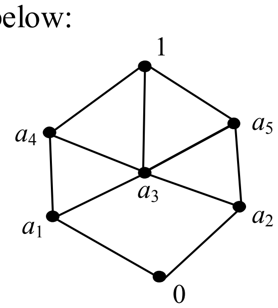
$\delta(q_1, b, b)=\{(q_1, b)\}$

$\delta(q_1, a, b)=\{(q_2, \lambda)\}$

(b) Let $\Sigma=\{a, b\}$ be an alphabet. Show that the language $L=\{a^n b^n : n \geq 1\}$ is not a regular language but it is a CFL. 6

15.(a) Prove that a lattice $L$ is non-distributive iff $N_5 \rightarrowtail L$ OR $M_3 \rightarrowtail L$. Here $L_1 \rightarrowtail L_2$ means $L_2$ contains a sublattice isomorphic to $L_1$. 8

(b) Consider the lattice $L$ given below: 4




Which of the following are sublattices of $L$?

$L_1=\{0, a_1, a_2, 1\}$ , $L_2=\{0, a_1, a_5, 1\}$

Justify your answer.

16.(a) Draw a switching circuit which realizes the following Boolean expressions: 3+3

(i) $x(yz+y'z')+x'(yz'+y'z)$

(ii) $(x+y+z+u)(x+y+u)(x+z)$

(b) For $n \in \mathbb{N}$, suppose $D_n$ denotes the set of all positive divisors of $n$. Then prove that $(D_n, \preccurlyeq)$ is a Boolean lattice iff $n$ is square free. Here, $a \preccurlyeq b$ iff $a \mid b$. Here for $a \in D_n$, $a'=\frac{n}{a}$. 6

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