'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2023

UMATMAJ11001-MATHEMATICS

CLASSICAL AND LINEAR ALGEBRA

Time Allotted: 2 Hours 30 Minutes | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) Find the conditions that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in G.P. 3

(b) Find the real part of $(1+i\sqrt{3})^{1+i}$. 3

(c) Prove that $n\,! > n^{\frac{n}{2}}$ $(n>1)$. 3

(d) Applying Descarte's rule of signs, find the nature of the roots of the equation 3

$$x^6 + x^4 + x^2 + x + 3 = 0$$

(e) If the amplitude of the complex number $\frac{z-1}{z+1}$ is $\frac{\pi}{4}$, show that z lies on a fixed circle with centre i. 3

(f) Find the characteristic equation and eigenvalues of the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. 3

GROUP-B

2. Answer any ***four*** questions: $6\times4 = 24$

(a) Find the range of values of k for which the equation $x^4 - 26x^2 + 48x - k = 0$ has four unequal roots. 6

(b) Solve the equation $x^3 - 6x - 4 = 0$ by Cardan's method. 6

(c) If $a, b, c, d > 0$ and $a+b+c+d = 1$, prove that 6

$$\frac{a}{1+b+c+d} + \frac{b}{1+a+c+d} + \frac{c}{1+a+b+d} + \frac{d}{1+a+b+c} \geq \frac{4}{7}$$

(d) If $u + iv = \tan(x+iy)$, then show that 6

(i) $u^2 + v^2 = 1 - 2u\cot 2x$; (ii) $u^2 + v^2 + 2v\left\{\frac{e^{-2y} + e^{2y}}{e^{-2y} - e^{2y}}\right\} + 1 = 0$

(e) Show that eigenvalues of a real symmetric matrix are all real. 6

(f) Determine the conditions for which the system of Linear equations: 6

$$x + 2y + z = 1$$
$$2x + y + 3z = b$$
$$x + ay + 3z = b + 1$$

Has (i) only one solution, (ii) no solution, (iii) many solutions.

GROUP-C

3. Answer any ***two*** questions: 12×2 = 24

(a) (i) Solve the equation $x^4-5x^3+11x^2-13x+6=0$, given that two of its roots α and β are connected by the relation $3\alpha+2\beta=7$. 6

(ii) If α, β, γ be the roots of $x^3+px^2+qx+r=0$ find the equation whose roots are $\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}$ 6

(b) (i) Let n be a positive integer. Prove that 4

$$\frac{1}{\sqrt{4n+1}}<\frac{3\cdot7\cdot11\cdots\cdots(4n-1)}{5\cdot9\cdot13\cdots\cdots(4n+1)}<\sqrt{\frac{3}{4n+3}}$$

(ii) If a, b, c are positive real numbers and $abc=k^3$, prove that 3

$$(1+a)(1+b)(1+c)\geq(1+k)^3$$

(iii) Using Strum's theorem, find the subintervals of $(-4, 3)$ in which the roots of equation $x^4-12x^2+12x-3=0$ lie. 5

(c) (i) State Cayley-Hamilton theorem. Using this theorem, find A^{-1}, where 1+5

$$A=\begin{pmatrix}1 & 2 & 1\\ 1 & -1 & 1\\ 2 & 3 & -1\end{pmatrix}$$

(ii) Examine if the matrices A and B are congruent, where 6

$$A=\begin{pmatrix}2 & -2 & 0\\ -2 & 1 & -2\\ 0 & -2 & 0\end{pmatrix}, \quad B=\begin{pmatrix}3 & 4 & 1\\ 4 & 9 & 4\\ 1 & 4 & 2\end{pmatrix}$$

(d) (i) For the matrix $A=\begin{pmatrix}1 & 0 & 1\\ 2 & 1 & 3\\ -1 & 1 & 2\end{pmatrix}$, find non-singular matrices P and Q such that PAQ is the fully reduced normal form. 6

(ii) If λ is an eigenvalue of a non-singular matrix A, then prove that λ^{-1} is an eigenvalue of A^{-1}. 2

(iii) Find row-equivalent row-reduced echelon matrix to the matrix. 4

$$\begin{bmatrix}1 & -1 & 2 & 0\\ 2 & 2 & 1 & 5\\ 1 & 3 & -1 & 0\\ 1 & 7 & -4 & 1\end{bmatrix}$$

and hence find its rank.

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