'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Sec 1st Semester Examination, 2023

UMATSEC11001-MATHEMATICS

LOGIC, INTEGERS, AND BOOLEAN ALGEBRA

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions: $1\times5 = 5$

(a) Represent the following expression as a switching circuits — 1

$$A(BC' + B'C) + A'BC$$

(b) For a prime p and a positive integer b, show that either p divides b or $\gcd(b,\ p) = 1$. — 1

(c) If $a \equiv b^2 (\mathrm{mod}\, 7)$, where a and b are any two given integers show that $7 \mid a^4 - b^8$. — 1

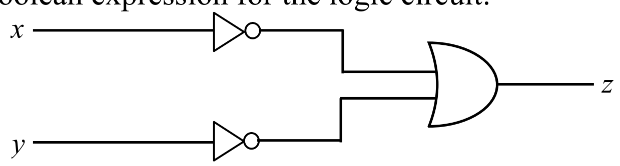
(d) Prove that the following proposition is tautology: — 1

$$\sim(p \wedge q) \vee q$$

(e) If p is true and q is false, find the truth values of the following: — 1

$$(p \wedge q) \rightarrow (p \vee q)$$

(f) Find the Boolean expression for the logic circuit. — 1



(g) Write the negation of the following statement: — 1

"If it is raining the game stands cancel"

(h) Find $\gcd(-100,\ 246)$. — 1

GROUP-B

2. Answer any ***three*** questions: $5\times3 = 15$

(a) Solve the linear congruence $9x \equiv 12 \ (\mathrm{mod}\ 15)$. — 5

(b) Use the principles of mathematical induction to prove that $(3+\sqrt{7})^n + (3-\sqrt{7})^n$ is an even integers for all $n \in \mathbb{N}$. — 5

(c) Use Karnaugh map to simplify $X = A'B'C' + A'BC' + AB'C$. — 5

(d) Use the theory of congruence to prove that $7 \mid 2^{5n+3} + 5^{2n+3}$ for all $n \geq 1$. 5

(e) Using a truth table to show that the following is a tautology: 5

$$((P \vee Q) \wedge (P \to R) \wedge (Q \to R)) \to R$$

GROUP-C

Answer any *two* questions 10×2 = 20

3. (a) Solve the following system of congruences 4

$$X \equiv 3 \pmod 7$$
$$X \equiv 5 \pmod 9$$
$$X \equiv 4 \pmod 5$$

(b) Prove that $\sim (p \wedge q) \to (\sim p \vee (\sim p \vee q)) = -p \vee q$. 4

(c) Convert the following Boolean function: 2

$f(x,\, y) = x \cdot y' + x' \cdot y + x' \cdot y'$ to maxterm expression (CNF)

4. (a) Draw a circuit which realize the Boolean function $f(x,\, y,\, z) = (x+y)\cdot(y+z)\cdot(z+x)$. Use the laws of Boolean algebra to show that the above circuit is equivalent to a switching circuit in which if any two switches are on, the light is on. Construct the equivalent switching circuit. 5

(b) Prove that $ab \equiv ac \pmod m \Leftrightarrow b \equiv c \left(\bmod \dfrac{m}{\gcd(a,\, m)} \right)$. 3

(c) Use congruence to show that 35078571 is divisible by 9. 2

5. (a) For any two element a and b in a Boolean algebra B, show that $(a \cdot b)' = a' + b'$. 3

(b) For any integer n, show that $7n+1$ and $15n+2$ are relatively prime. 2

(c) Use the Quine-McCluskey algorithm to find the prime implicants of the following expression. Also find the minimal expression of the function 5

$$f(a,\, b,\, c) = \sum m(0,\, 2,\, 3,\, 7)$$

6. (a) Write down an equivalent form of $P \wedge (Q \leftrightarrow R) \vee (R \leftrightarrow P)$, which does not contain a biconditional operator. 3

(b) (i) State Euclidean Algorithm. Use it to find $\gcd(119,\, 272)$. 3

(ii) Prove that $a \equiv b \pmod m \Leftrightarrow a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$, where $m = m_1 m_2$ and $\gcd(m_1,\, m_2) = 1$.

(c) Translate each of the following into logical expression using predicates, quantifiers and logical connectivities. 4

(i) No Physics student know C++.

(ii) All Mathematics students know C++.

(iii) At least one Mathematics student know C++.

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