'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2023

UPHYMAJ11001-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions from the following: 1×5 = 5

(a) $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{k}$ are three vectors such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$. Find the value of λ.

(b) What is the greatest rate of increase of $u = xyz^2$ at (1, 0, 3)?

(c) Find out whether $\sin(\omega t)$ and $\cos(\omega t)$ can be two solutions of a second order homogeneous ordinary differential equation.

(d) State Stokes' theorem in vector analysis.

(e) How many solutions the ordinary differential equation $\frac{2}{3}\frac{dy}{dx} = y^{1/3},\ y(0) = 0$ have?

(f) If $\oint_C \vec{A} \cdot d\vec{r} = 0$, is the vector field $\vec{A}$ solenoidal or irrotational?

(g) What is the expression of a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$?

(h) Find the degree of the differential equation: $\left(\frac{d^2y}{dx^2} + 2\right)^{3/2} = x\left(\frac{dy}{dx}\right)$

GROUP-B

Answer any *three* questions from the following 5×3 = 15

2. (a) Find a second order homogeneous differential equation, whose solutions are $1, e^{-2x}$. 2

(b) Solve the differential equation $2xy\frac{dy}{dx} = x^2 + y^2$. 3

3. (a) Find the directional derivative of the divergence of $\vec{F} = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ at the point (3, 1, 4) in the direction of outwardly directed normal to the sphere $x^2 + y^2 + z^2 = 4$. 3

(b) Evaluate $\vec{\nabla} \times (\phi\,\vec{\nabla}\phi)$. 2

4. (a) Prove that spherical co-ordinate system is orthogonal. 3

(b) Prove that $\nabla^2(\ln r) = \frac{1}{r^2}$, where $\vec{r}$ is the position vector. 2

5. Represent the vector $\vec{F} = 2y\hat{i} - 3x\hat{j} + x^2\hat{k}$ in cylindrical co-ordinate system. Thus determine F_ρ, F_ϕ and F_z. 5

6. (a) Find a vector normal to a plane consisting of points $P_1(0, 1, 0)$, $P_2(1, 0, 1)$ and $P_3(0, 0, 1)$. 2

(b) If $\vec{F} = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve $y^2 = x$, from (0, 0) to (1, 1). 3

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. (a) Find the integrating factor of the differential equation: 2+2

$$x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$$

Hence find the general solution.

(b) Solve the differential equation: 4

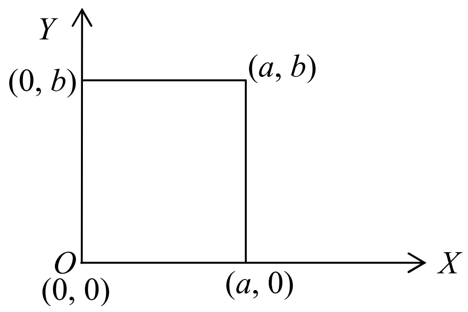
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^3 + x$$

(c) If $\hat{e}$ is an unit vector, show that $\vec{\nabla}(\hat{e} \cdot \vec{r}) \cdot \hat{e} = 1$. 2

8. (a) Verify Gauss's divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 4

(b) Evaluate $\iint_S \vec{A} \cdot \hat{n}\, ds$ where, $\vec{A} = z\hat{i} - x\hat{j} - 3y^2 z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. 6

9. (a) Find the value of $\oint_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the boundary of the rectangle shown in fig. 5



(b) A given vector is $\vec{u} = \frac{1}{3}(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k})$ and $\hat{n}$ is the unit normal vector to the surface of the hemisphere $(x^2 + y^2 + z^2 = 1;\ z \geq 0)$. Show that the value of the integral $\int_S (\vec{\nabla} \times \vec{u}) \cdot \hat{n}\, ds$ evaluated on the curved surface S of the hemisphere is $\frac{\pi}{2}$. 5

10.(a) Determine volume from $\int_{x=2}^{3} \int_{y=1}^{2} \int_{z=0}^{1} 8xyz\, dv$. 2

(b) Show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2}\oint_C (x\, dy - y\, dx) = \oint_C x\, dy = -\oint_C y\, dx$. Hence calculate the area of the ellipse $x = a\cos\phi$, $y = b\sin\phi$. 3+2

(c) If $\vec{r}(t)$ be a vector of fixed magnitude, show that $\frac{d\vec{r}(t)}{dt}$ is perpendicular to $\vec{r}(t)$. 3

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