UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2021

MATHEMATICS

PAPER-V

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

Answer *all* questions

GROUP-A

1. (a) Use Cauchy's Principle of convergence to prove that the series $\sum \frac{1}{n}$ does not converge. 3

   (b) Show that the series $\sum \frac{1}{\sqrt{n}} \tan \frac{1}{n}$ is convergent. 4

   (c) Test the convergence of the series $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+\cdots\cdots$. 3

2. (a) Let $f:[a,\, b] \longrightarrow \mathbb{R}$ be continuous on $[a,\, b]$ and $x_1,\, x_2,\, x_3 \in [a,\, b]$. Prove that there is a point $c \in [a,\, b]$ such that $f(c)=\frac{f(x_1)+f(x_2)+f(x_3)}{2!}$. 3

   (b) A function $f:[a,\, b] \longrightarrow \mathbb{R}$ is said to be satisfy a Lipschitz condition of order α on $[a,\, b]$ if there exists a real number $M>0$ such that $|f(x)-f(y)|<M|x-y|^{\alpha}$ for all $x,\ y \in [a,\, b]$. 3

   If f satisfies a Lipschitz condition of order $\alpha > 1$ on $[a,\, b]$ prove that f is a constant function on $[a,\, b]$.

   (c) Find the points of local maximum and local minimum of the function 4

$$f(x)=8x^5-10x^3+5x^2+1\,, \quad x \in \mathbb{R}$$

GROUP-B

3. (a) Show that for the function $f(x,\, y)=\begin{cases} \dfrac{x^2y^2}{x^2+y^2} & (x,\, y) \neq (0,\, 0) \\ 0 & \text{if } x=y=0 \end{cases}$ 6

   $f_{xy}(0,\, 0)=f_{yx}(0,\, 0)$, even through the conditions of Schwarz's theorem and also of Young's theorem are not satisfied.

(b) The roots of the equation in α, $(\alpha-x)^3+(\alpha-y)^3+(\alpha-z)^3=0$ are $u,\ v,\ w$. 4

Prove that $\dfrac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)}=-2\dfrac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$.

GROUP-C

4. (a) Find the interval on which the function is concave up or concave down, 5

$$g(x)=x^3-3x^2-9x+1$$

(b) If $I_n=\int_0^{\pi/2} x\sin^n x\,dx$ and $n>1$ then, prove that $I_n=\dfrac{n-1}{n}I_{n-2}+\dfrac{1}{n^2}$. 5

5. (a) Given $g(x)=\dfrac{3x^2+4x-3}{x^2+3}$, determine the horizontal asymptote and the point where the graph crosses the horizontal asymptote. 5

(b) Find the stationary points of the function $y=2x^3-9x^2+12x-3$ and determine their nature. 5

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