UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2021

MATHEMATICS

PAPER-XI

Full Marks: 50

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

Answer *all* the questions 10×5 = 50

GROUP-A

1. (a) Let (X, d) be a metric space. Determine the constant k such that $d+k$ is also a metric on X. 2

   (b) Show that in a discrete metric space every subset is open as well as closed. 3

   (c) Find the boundary of the set $\{2+\frac{1}{n} : n \in \mathbb{N}\}$ in $\mathbb{R}$ with the usual metric. 2

   (d) Let (X, d) be a metric space. Prove that, for $x, y, z \in X$, 3

$$|d(x, y)-d(z, w)| \le d(x, z)+d(y, w)$$

2. (a) Let $(\mathbb{R}, d)$ be the usual metric space. Show that the set of all integers is a complete metric space in $(\mathbb{R}, d)$. 3

   (b) Let X denote the set of all sequences of real numbers. If $x=(x_n)$ and $y=(y_n)$ are two elements of X, then show that 5

$$f(x, y)=\sum_{i=1}^{\infty}\frac{1}{2^i}\min\{|x_n-y_n|, 1\}$$

   is a metric on X.

   (c) Define a metric on $\mathbb{R}$ such that $\frac{1}{n}\to 5$ but $-\frac{1}{n}\to 0$. 2

GROUP-B

3. (a) Find the image of the point $\frac{1-i}{2}$ on the Riemann sphere $x^2+y^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}$. 2

   (b) Find the bilinear transformation which maps the points $z=\infty, 1, 0$ into $w=0, i, \infty$. 2

(c) Let f be an analytic function in a domain D. If $\arg f(z)$ is constant for $z \in D$, then show that f must be constant. 2

(d) Find the analytic function $f(z)$, whose real part is 4

$$e^{-x}\{(x^2 - y^2)\cos y + 2xy\sin y\}$$

4. (a) For what values of z, $f(z) = \bar{z}$ satisfies the C-R equations? 2

(b) Show that the stereographic projections of the points z and $-\frac{1}{z}$ are diametrically opposite points on the Riemann sphere. 3

(c) Show that $f(z) = xy + iy$ is continuous everywhere but not analytic, where $z = x + iy$. 2

(d) Prove that $\frac{d}{dz}(\cos z) = -\sin z$ and $\frac{d}{dz}(\sin z) = \cos z$. 3

GROUP-C

5. (a) Prove that the groups $(\mathbb{R} - \{0\}, \times)$ and $(\mathbb{R}, +)$ are not isomorphic. 3

(b) Prove that a group G is abelian if $x^2 = 1, \forall x \in G$. 2

(c) Let $(G, \circ)$ be a group and a mapping $\varphi : G \to G$ is defined by $\varphi(x) = x^{-1}$, $x \in G$. Prove that φ is a homomorphism iff G is commutative. 2

(d) Let G be a group in which $(ab)^3 = a^3b^3$ for all $a, b \in G$. Prove that $H = \{x^3 : x \in G\}$ is a normal subgroup of G. 3

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