'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2022

MATHEMATICS

PAPER-V

REAL ANALYSIS, CAL. OF SEVERAL VARIABLES, APP. OF CALCULUS

NEW SYLLABUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer Question No. 1 and any *two* from the rest

1. (a) Show that $\lim\limits_{x\to 0}\sin(\frac{1}{x})$ does not exist. 2

(b) Give an example of a function $f:\mathbb{R}\to\mathbb{R}$ which is continuous only at $x=1$. 2

(c) Show that $2<e<3$. 2

2. (a) Prove that $\lim\limits_{n\to\infty}\left\{\frac{(n+1)(n+2)\cdots 2n}{n}\right\}^{1/n}=4/e$. 3

(b) Evaluate: $\lim\limits_{x\to 3}\left([x]-\left[\frac{x}{3}\right]\right)$ 3

(c) Let $D\subset\mathbb{R}$ and $f:D\to\mathbb{R}$ be a function. If c be an isolated point of D, then f is continuous at c. 1

3. (a) Prove that $\left(1+\frac{1}{x}\right)^x>\left(1+\frac{1}{y}\right)^y$, where $x,\ y\in\mathbb{R}$ and $x>y>0$. 3

(b) State and prove Rolle's theorem. 4

4. (a) Let $f(x)=x^5-5x^4+5x^3+10$, $x\in\mathbb{R}$. Show that f has no extremum at $x=0$. 3

(b) It is desired to make an open box with square base out of a square piece of cardboard of side 1 foot by cutting equal squares out of the corners and then folding up the cardboard to form the sides of the box. What must be the length of the side of the squares cut out in order to maximize the volume? 4

GROUP-B

Answer any *one* question

5. (a) State and prove Young's theorem. 5

(b) Consider the function $f(x,y)=\begin{cases}\dfrac{x^3y}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ 3

Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

(c) Explain the result $(5/b)$ in light of Young's theorem. 2

6. (a) Consider the function 3

$$f(x, y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2} & ;\ (x, y)\neq(0, 0) \\ 0 & ;\ (x, y)=(0, 0) \end{cases}$$

Show that $f_{xy}(0, 0)\neq f_{yx}(0, 0)$.

(b) State Schwartz's theorem. 2

(c) Explain the result $[6/a]$ in light of Schwartz's theorem. 2

(d) Let $F_1=f_1(x_1)$, $F_2=f_2(x_1, x_2)$, $\cdots$, $F_n=f_n(x_1, x_2, \cdots, x_n)$, then show that the Jacobian $\dfrac{\partial(F_1, F_2, \cdots, F_n)}{\partial(x_1, x_2, \cdots, x_n)}=\dfrac{\partial F_1}{\partial x_1}\cdot\dfrac{\partial F_2}{\partial x_2}\cdot\cdots\cdot\dfrac{\partial F_n}{\partial x_n}$. 3

GROUP-C

Answer Question no. 7 and any *two* from the rest

7. (a) Find the pedal equation of the parametric curve $x=a\cos^3\theta$; $y=a\sin^3\theta$. 2

(b) Find the centre of gravity of a uniform circular ring. 2

(c) Evaluate: $\int \sin^5 x\, dx$ 2

8. (a) If $m\in\mathbb{Q}^+$ and $n\in\mathbb{N}$ and $I_{m,n}=\int x^m(\log x)^n\, dx$, show that 4

$$I_{m,n}=(\log x)^n\cdot\frac{x^{m+1}}{m+1}-\frac{n}{m+1}I_{m,n-1}$$

(b) Evaluate: $\int x^{3/2}(\log x)^3\, dx$ 3

9. (a) If ρ_1 and ρ_2 are the radii of curvature at the extremities of any chord of the cardioid $r=a(1+\cos\theta)$, which passes through the pole, prove that 5

$$\rho_1^2+\rho_2^2=\frac{16}{9}a^2$$

(b) Find all the asymptotes of $x^3-2x^2y+xy^2+x^2-xy+2=0$. 2

10.(a) The area enclosed by the astroid 2

$$x^{2/3}+y^{2/3}=a^{2/3}$$

is revolved about x-axis. Find the volume of the solid generated.

(b) Find the C.G. (centre of gravity) of the solid formed by the revolution of the quadrant of the ellipse 4

$$x^2/a^2+y^2/b^2=1$$

about its major axis.

(c) State Pappus theorem. 1

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