'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2022

MATHEMATICS

PAPER-VI

INTEGRAL CALCULUS, DYNAMICS OF PARTICLES

NEW SYLLABUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

(Marks:10)

Answer any *two* questions

1. If a bounded function f on $[a, b]$ has a set of finite points of discontinuities on $[a, b]$, then prove that f is R-integrable on $[a, b]$. 5

2. State and prove the fundamental theorem of integral calculus. 1+4

3. (a) Show that $\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}} < \frac{\pi}{6}$. 3

(b) Verify the first mean value theorem for integral calculus of the following functions: 2

$$f(x) = x \quad , \quad g(x) = x^2 \quad \forall\, x \in [-1,\ 1]$$

GROUP-B

(Marks:40)

Answer Questions No. 4 and any *three* from the rest

4. Answer any ***two*** questions: 2×2 = 4

(a) The law of motion in a straight line being $x = \frac{1}{2}vt$. Show that the acceleration is constant. 2

(b) If a particle moves in a circle, then prove that its angular velocity about a point on a circumference is half of its angular velocity about its centre. 2

(c) A particle describes an equiangular spiral $r = ae^{\theta}$ in such a manner that the radial acceleration is zero. Prove that the velocity of the particle is proportional to r. 2

5. (a) Two bodies of masses M and M' are attached to the lower end of an elastic string whose upper-end is fixed and hang at rest; M' falls off; show that the distance of M from the upper-end of the string at time t is 6

$$a + b + c\cos\left(\sqrt{\frac{g}{b}}t\right)$$

where a is the unstretched length of the string, b and c are the distances by which it would be stretched when supporting M and M' respectively.

(b) A particle of mass m falls from the rest towards a centre of force varying inversely as the square of the distance from the centre. Show that the time of descend through the first half of its initial distance is to that through the last half as $(\pi+2):(\pi-2)$. 6

6. (a) Find the tangential and normal components of velocity and acceleration of a particle which describes a plane curve. 6

(b) A particle moves in the curve $y = a\log\left\{\sec\left(\frac{x}{a}\right)\right\}$ in such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature. 6

7. (a) A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane under the action of central force F per unit mass. Find the differential equation of its path in the form: 6

$$F = \frac{h^2}{p^3}\frac{dp}{dr}$$

(b) A particle of mass m moves under a central attractive force $m\mu(5u^3 + 8c^2u^5)$ and is projected from an apse at a distance c with velocity $\frac{3\sqrt{\mu}}{c}$. 4+2

Prove that the orbit of the particle is $r = c\cos\frac{2}{3}\theta$ and that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{\mu}}$.

8. (a) A particle is projected vertically upwards under gravity with a velocity V. Assuming that the resistance of air is kv per unit mass, where v is the velocity of the particle and k is a constant. Obtain the equation of motion of the particle and show that the particle comes to rest at a height 1+5

$$\frac{V}{k} - \frac{g}{k^2}\log\left(1 + \frac{kV}{g}\right)$$

above the point of projection.

(b) A bead moves along a rough curved wire which is such that it changes its direction of motion with constant angular velocity. Show that a possible form of the wire is an equiangular spiral. 6

9. (a) A falling rain drop has its radius uniformly increased by gathering moisture. If it is given a horizontal velocity $2\lambda a$, show that it will describe a hyperbola, one of whose asymptotes is vertical. 6

(b) If a planet was suddenly stopped in its orbit supposed circular, show that it would fall into the sum in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 6

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