'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2022

MATHEMATICS

PAPER-VII

MODERN ALG, LINEAR ALG, VECTOR ANALYSIS

NEW SYLLABUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

(Marks:15)

Answer Question No. 1 and any *one* from the rest

1. (a) Give an example of an abelian group which is not cyclic. 1

(b) Find the order of the permutations 2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 5 & 1 & 2 \end{pmatrix}$$

(c) Examine if the ring of matrices $\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ is a field. 2

2. (a) State and prove Lagrange's theorem. 5

(b) Describe all even permutation on the set $\{1, 2, 3, 4\}$. 5

3. (a) Define a field. Give an example of a ring which is not a field. Show that a field does not contain any divisor of zero. 2+2+2

(b) If a is a generator of a cyclic group, show that a^{-1} is also a generator of it. 4

GROUP-B

(Marks:15)

Answer Question No. 4 and any *one* from the rest

4. Answer any ***two*** questions:

(a) Let $T: R^3 \to R$; where $T(1, 1, 1) = 3$, $T(0, 1, -2) = 1$, $T(0, 0, 1) = -2$. Find $T(x, y, z)$. $2\frac{1}{2}$

(b) Test whether the mapping $T:\ \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = xy$ are linear or not. $2\frac{1}{2}$

(c) Prove that for vectors $\vec{\alpha}, \vec{\beta}$ is an Euclidean space V, $\langle \vec{\alpha}, \vec{\beta} \rangle = 0$ iff $2\frac{1}{2}$

$$\| \vec{\alpha} + \vec{\beta} \| = \| \vec{\alpha} - \vec{\beta} \|$$

5. (a) A linear mapping $T: R^3 \to R^3$ is defined by $T(x, y, z) = (x - y,\ x + 2y,\ y + 3z)$. Show that T is non-singular and determine T^{-1}. 5

(b) State and prove Bessel's inequality. 5

6. (a) A linear transformation $T: V_n(F) \to V_n(F)$ is non-singular iff for every set $\{v_1, v_2, \cdots, v_n\}$ linearly independent in V, the set $\{T(v_1), T(v_2), \cdots, T(v_n)\}$ is linearly independent in V. 5

(b) Let P be a subspace of a finite dimensional Euclidean space V. Then prove that $V = P \oplus P^{\perp}$. 5

GROUP-C

(Marks:20)

Answer Question No. 7 and any *three* from the rest

7. (a) If $u = x^3 + 3yz^2$, then find ∇u. 1

(b) Show that $\vec{\nabla} f(x, y, z)$ is both an irrational and solenoidal vector, if $f(x, y, z)$ satisfies $\vec{\nabla}^2 f(x, y, z) = 0$. 2

(c) Find the directional derivative of $f = xy + yz + zx$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point $(1, 2, 0)$. 2

8. Show that the vector field $\vec{f} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Find $\phi(x, y, z)$ such that $\vec{f} = \vec{\nabla}\phi$. 5

9. Evaluate the surface integral $\int_S (yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k})\,dS$; where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 5

10. The necessary and sufficient condition that the vector field defined by the vector point function $\vec{F}$ with continuous derivative be conservative is that $\vec{\nabla} \times \vec{F} = 0$. 5

11. Verify the divergence theorem for the vector function $\vec{F} = 2xz\vec{i} + y^2\vec{j} + yz\vec{k}$ taken over the surface of the curve bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 5

12. Verify Green's theorem in a plane for $\oint_C \{(x^2 + xy)\,dx + x\,dy\}$, where C is the curve enclosing the region bounded by $y = x^2$ and $y = x$. 5

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