'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2022

MATHEMATICS

PAPER-VIII

GEOMETRY, DIFFERENTIAL EQUATION

NEW SYLLABUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer the following questions:

   (a) Find the direction cosines of the line, which is equally inclined to the axes. 1

   (b) Find the equation of the sphere passing through the points (2, 0, 0), (0, 2, 0), (0, 0, 2) and having the least possible radius. 2

   (c) Determine the value of K so that the lines 2

   $$\frac{x-1}{2}=\frac{y-4}{1}=\frac{z-5}{2} \quad \text{and} \quad \frac{x-2}{-1}=\frac{y-8}{K}=\frac{z-11}{4}$$

   may intersect.

2. Answer any ***two*** of the following questions: 4×2 = 8

   (a) Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ on the plane $x+2y+z=6$. 4

   (b) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z-8=0$ is a great circle. 4

   (c) The plane $lx+my=0$ is rotated about its line of intersection with the plane $z=0$ through an angle α. Prove that its equation in its new position is 4

   $$lx+my\pm z\sqrt{l^2+m^2}\tan\alpha=0$$

3. Answer any ***two*** of the following questions: 6×2 = 12

   (a) Show that the condition that the plane $ax+by+cz=0$ may cut the cone $yz+zx+xy=0$ in perpendicular lines is $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 6

(b) Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1,\ x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1,\ y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if $2d$ is the s.d., prove that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$. 6

(c) Show that $2x^2+5y^2+5z^2+4xy+2yz-4zx+16x+22y-10z-18=0$ is the equation of the cylinder which passes through the point $(3,-1,-1)$ and has the axis $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-2}{1}$. 6

GROUP-B

4. Answer the following questions:

(a) Find the order and degree of the partial differential equation 1

$$y\left\{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right\}=z\frac{\partial z}{\partial y}$$

(b) Form the partial differential equation by elimination of ϕ from 2

$$lx+my+nz=\phi(x^2+y^2+z^2)$$

(c) Show that $x=0$ is an ordinary point of $(x^2-1)y''+xy'-y=0$, but $x=1$ is a regular singular point. 2

5. Answer any ***two*** of the following questions: 4×2 = 8

(a) Reduce the equation 4

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+4x^2y=e^{x^2}$$ to its normal form and hence solve it.

(b) Solve: $Dx-y=0\ ;\ (D-1)x-(D+1)y=0$, where, $D=\frac{d}{dt}$. 4

(c) Find a complete integral of $zpq=p+q\ ,\quad p=\frac{\partial z}{\partial x}\ ,\quad q=\frac{\partial z}{\partial y}$. 4

6. Answer any ***two*** of the following questions: 6×2 = 12

(a) Solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$, by method of variation of parameters. 6

(b) Find the series solution of the differential equation 6

$$2x^2y''-xy'+(1-x^2)y=0 \quad \text{about} \quad x=0$$

(c) Find the eigen values and eigen functions of 6

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0\ \ (\lambda>0)\quad ;\quad y(1)=0\quad ,\quad y(e^{\pi})=0$$

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