'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2022

MATHEMATICS

PAPER-IX

LINEAR PROGRAMMING AND OPTIMIZATION, TENSOR ALGEBRA AND ANALYSIS

NEW SYLLABUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

Answer Question No. 1 and any *two* from the rest

1. Answer any ***three*** questions from the following: $2\times3 = 6$

(a) Show that the set $X=\{x:\ |x|\le 2\}$ is a convex set in E. 2

(b) Find dual of the following L.P.P.: 2

Maximize $Z=4x_1+3x_2$

Subject to, $x_1+x_2\le 5$

$2x_1-3x_2\le 2$

$x_1,\ x_2\ge 0$

(c) Determine the extreme points of the set $S=\{(x_1,\ x_2):\ 0\le x_1\le 2,\ 1\le x_2\le 3\}$. 2

(d) Find graphically the feasible space, if any, for the following problem: 2

$2x_1+3x_2\le 6$

$2x_1+3x_2\ge 6$

$x_1,\ x_2\ge 0$

(e) In the following equations, find the basic solution with x_3 as the non-basic variable: 2

$x_1+4x_2-x_3=3$

$5x_1+2x_2+3x_3=4$

2. (a) Use duality to find the optimal solution, if any, of the L.P.P.: 6

Minimize $Z=10x_1+6x_2+2x_3$

Subject to, $-x_1+x_2+x_3\ge 1$

$3x_1+x_2-x_3\ge 2$

$x_1,\ x_2,\ x_3\ge 0$

(b) Show that, the number of basic variables in a transportation problem is at most $(m+n-1)$. 6

3. (a) Use two-phase method to solve the following L.P.P.: 6

Maximize $Z = 3x_1 - x_2$

Subject to, $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_2 \leq 4$

$x_1,\ x_2 \geq 0$

(b) Solve graphically the following L.P.P.: 6

Minimize $Z = -x_1 + 2x_2$

Subject to, $-x_1 + 3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1,\ x_2 \geq 0$

4. (a) Prove that $x_1 = 2,\ x_2 = 3,\ x_3 = 0$ is a feasible solution but not a basic feasible solution of the set of equations. 4+4

$$3x_1 + 5x_2 - 7x_3 = 21$$

$$6x_1 + 10x_2 + 3x_3 = 42$$

Find the basic feasible solution of the above set of equations.

(b) Find the dual of the following L.P.P.: 4

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to, $4x_1 + 3x_2 + x_3 = 6$

$x_1 + 2x_2 + 5x_3 = 4$

$x_1,\ x_2,\ x_3 \geq 0$

5. (a) Solve the assignment problem with the following cost matrix: 6

	M_1	M_2	M_3	M_4	M_5
I	9	8	7	6	4
II	5	7	5	6	8
III	8	7	6	3	5
IV	8	5	4	9	3
V	6	7	6	8	5

(b) Solve the following travelling salesman problem: 6

	A	B	C	D
A	∞	12	10	15
B	16	∞	11	13
C	17	18	∞	20
D	13	11	18	∞

GROUP-B

Answer Question No. 6 and any *three* from the rest

6. (a) Define covariant and contravariant vector. 1

(b) Prove that $\delta^i_j \, \delta^j_k = \delta^i_k$. 2

(c) If A_{ij} is a symmetric tensor and $B_{ij} = A_{ji}$, show that B_{ij} is a symmetric tensor. 2

7. If $a_{ij}\, u^i u^j$ is an invariant, where u^i is an arbitrary contravariant vector, a_{ij} is a symmetric tensor and $u^i = A^i + B^i$, then show that $a_{ij} A^i B^j$ is an invariant. 5

8. If the relation $b^{ij} u_i u_j = 0$ holds for any arbitrary covariant vector u_i, prove that $b^{ij} + b^{ji} = 0$. 5

9. In a 4-dimensional space-time $(x,\ y,\ z,\ ct)$, the line element is 5

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$$

then show that $(\sqrt{2},\ 0,\ 0,\ \sqrt{3}/c)$ is a unit vector.

10. If A_i is a covariant vector, prove that $\left(\dfrac{\partial A_i}{\partial x^j} - \dfrac{\partial A_j}{\partial x^i}\right)$ is a covariant tensor of rank 2. 5

11. Prove that, $A^{ij}_{,j} = \dfrac{1}{\sqrt{g}} \dfrac{\partial}{\partial x^j}(A^{ij}\sqrt{g}) + A^{jp}\begin{Bmatrix} i \\ j \ \ p \end{Bmatrix}$, where A^{ij} is a tensor of type $(2, 0)$, symbols have their usual meaning. 5

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