'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2022

MATHEMATICS

PAPER-XI

METRIC SPACE, COMPLEX ANALYSIS, ALGEBRA

NEW SYLLABUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A (Full marks-20)

Answer Question number 1 and any *three* from the rest

1. (a) Give an example of an incomplete metric space. 1

   (b) Let (X, d) be a metric space. Find the value of k such that d_1 is a metric on X, where $d_1(x, y) = d(x, y) + k$; $\forall\, x, y \in X$. 2

   (c) In the Euclidean line, let $A = (0,\ 1]$, $B = (1,\ 2)$. Find $\inf(A \cup B)$. 2

2. Let $X = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\cdots, \frac{1}{n}, \cdots\cdots\right\}$ and 'd' is the usual metric defined on X. Let $A = \left\{1, \frac{1}{3}, \frac{1}{5}, \cdots\cdots, \frac{1}{2n-1}, \cdots\cdots\right\}$ and $B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \cdots\cdots\right\}$. Find distance between A and B. 5

3. Prove that every open sphere is neighbourhood of each of its points. 5

4. Prove that every Cauchy sequence in a metric space is convergent if and only if it has a convergent subsequence. 5

5. Prove that in a metric space (X, d), a subset A of X is closed iff A^c is open. 5

6. Show that $(\mathbb{R}^2, d)$ is a metric space where $d(x,\ y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ $\forall\ x, y \in \mathbb{R}^2,\ x = (x_1,\ x_2),\ y = (y_1,\ y_2)$. 5

GROUP-B (Full marks-20)

Answer Question number 7 and any *three* from the rest

7. (a) Show that the inverse of bilinear transformation is a bilinear transformation. 2

(b) Is the function $f(z)=xy+iy$ analytic everywhere? 2

(c) Define cross ratio of four points z_1, z_2, z_3 and z_4. 1

8. Let f be analytic function in a region G. If any one of $\text{Re}(f)$, $\text{Im}(f)$, $|f|$ is constant on G, then prove that f is constant on G. 5

9. Define stereographic projection. What is the image of the point $z=-\frac{1}{2}+i\frac{1}{2}$ on the sphere $x^2+y^2+z^2=1$ under stereographic projection? 5

10. Show that the composition of two bilinear transformation is a bilinear transformation. 5

11. Determine $f(z)=u+iv$ by obtaining a conjugate of a given harmonic function $u(x,\ y)=y^3-3x^2y$. 5

12. Let $f(z)=|z|^2$. Show that the derivative of $f(z)$ exists only at the origin. 5

GROUP-C

Answer any *two* questions 5×2 = 10

13. Prove that any two cyclic groups are isomorphic. 5

14. Let $G=(\mathbb{R},+)$, $G'=\{z\in\mathbb{C}\ :\ |z|=1\}$ and $\phi=G\to G'$ is defined by 5

$$\phi(x)=\cos 2\pi x+i\sin 2\pi x\quad ,\quad x\in\mathbb{R}$$

Prove that ϕ is a homomorphism. Determine $\ker\phi$.

15. A subgroup H of a group G is normal iff $xHx^{-1}=H$, $\forall\ x\in G$. 5

16. Suppose M and H are normal subgroups of a group G such that $M\cap H=\{e\}$. Then show that every element M commutes with every element of H i.e., $mh=hm$, where $m\in M$ and $h\in H$. 5

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