



'সমানো মন্ত্র: সমিতি: সমানী'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-III Examination, 2022

MATHEMATICS

PAPER-XII

THEORY OF PROBABILITY AND RIGID DYNAMICS

NEW SYLLABUS

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
All symbols are of usual significance.*

GROUP-A

Answer Question no. 1 and any three from the rest

1. (a) Define statistical regularity. 1
(b) Find the correlation coefficient of two regression lines $x + 6y = 6$ and $3x + 5y = 10$. 2
(c) If X be a Poisson variate such that $P(X = 1) = P(X = 2)$. Then find $P(X = 3)$. 2

2. (a) Consider a class of $\{X, Y, Z\}$ of events. Suppose it is known that $\{X, Y\}$ and $\{Y, Z\}$ are independent pairs of events. Does it follow that $\{X, Z\}$ is an independent pair? Justify your answer. 5
(b) The probability density function of a random variable X is given by 5
$$f(x) = C e^{-(x^2+2x+3)}, \quad -\infty < x < \infty$$
Find the constant C .

3. (a) If the random variables X and Y are uncorrelated and U and V are defined by 5
$$U = X \cos \alpha + Y \sin \alpha$$
$$V = -X \sin \alpha + Y \cos \alpha$$
then show that
$$\rho(U, V) = \frac{\sigma_y^2 - \sigma_x^2}{\sqrt{(\sigma_y^2 - \sigma_x^2) + 4\sigma_x^2\sigma_y^2 \operatorname{cosec}^2 2\alpha}}$$

(b) A rod of length 'a' is broken into three pieces at random. Find the probability of their forming a triangle. 5

4. (a) If the statistic X is a consistent estimate of α then prove that X^2 is also a consistent estimate of α^2 . 5
(b) State and prove Tchebycheff's inequality. 5
5. (a) Show that the moment generating function for uniform distribution over $(-a, a)$ is $\frac{\sinh(at)}{at}$. 5
(b) Define convergence in probability. If X_n be a binomial (n, p) variate, then show that $\frac{X_n}{n} \xrightarrow{\text{in } p} p$ as $n \rightarrow \infty$. 5
6. (a) In a lottery with 10,000 tickets there are 100 prizes. A man buys 100 tickets. Apply Poisson approximation to Binomial law to find the approximate probability of his winning at least one ticket. 5
(b) Using the method of likelihood ratio testing, describe a method of testing hypothesis $H_0 : \sigma = \sigma_0$ for a normal (μ, σ) population. 5

GROUP-B

Answer Question No. 7 and any one from the rest

7. (a) State the principle of conservation of energy. 2
(b) Define conservative system of forces. 1
(c) Write down the general equation of motion of a rigid body under impulsive forces. 2
8. (a) Two uniform rods, AB and AC are freely joined at A and are placed on a smooth table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to itself, show that the resulting velocities of the middle points of AB and AC are in the ratio 2 : 7. 5
(b) Define compound pendulum. Find the time of oscillation of a given compound pendulum. Hence, find the length of simple equivalent. 5
9. (a) Show that the kinetic energy of a body of mass M moving in two-dimension is driven by 5
- $$\frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\dot{\theta}^2$$
- (b) If an axis passes through the centre of gravity of a rigid body and is a principal axis at any point of its length, then show that it is a principal axis at all points of its length. 5

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