



'সমানো মন্ত্র: সমিতি: সমানী'

**UNIVERSITY OF NORTH BENGAL**  
B.Sc. Honours Part-III Examination, 2022

**MATHEMATICS**

**PAPER-XIII**

**THEORY OF STATISTIC, ANALYTICAL STATICS**

**NEW SYLLABUS**

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.  
All symbols are of usual significance.*

**GROUP-A**

**Answer Question No. 1 and any two from the rest**

1. (a) What do you mean by an efficient estimator? 1
- (b) If the lines  $4x + y = 52$  and  $x + y = 32$  be the regression lines of  $x$  on  $y$  and of  $y$  on  $x$  respectively, then find the correlation coefficient. 2
- (c) The mean and variance of a sample of size 400 from a normal population are found to be 18.35 and 3.25 respectively. Given  $P(U > 1.96) = 0.025$ ,  $U$  being a standard normal variate. Find 95% confidence interval for the population mean. 2
2. (a) Nine patients to whom a certain drug was administered, registered the following rise in blood pressure: 5  
 $3, 7, 4, -1, -1, 6, -4, 1, 5$   
Test the hypothesis that the drug does not raise blood pressure at 10% level of significance. Assume that the sample is from a normal population. Given  $P(t > 1.86) = 0.05$  for eight degrees of freedom.
- (b) Prove that the maximum likelihood estimate of the parameter  $\alpha$  of a population having density function 5  
$$\frac{2}{\alpha^2}(\alpha - x), \quad 0 < x < \alpha$$
for a sample of unit size is  $2x$ ,  $x$  being the sample value. Show also that the estimate is biased.
3. (a) For a large lot of freshly minted coins a random sample of size 50 is taken. The mean weight of coins in the sample is found to be 28.57 gm. Assuming that the population standard deviation of weight is 1.25 gm, will it be reasonable to suppose that the population mean is 28 gm? 5
- (b) Obtain the maximum likelihood estimate of the parameter  $p$  of the binomial  $(N, p)$  population for  $n$  sample values. 5

4. (a) The random variables  $X$  and  $Y$  are jointly normally distributed and  $U, V$  are defined by 5

$$U = X \cos \alpha + Y \sin \alpha,$$

$$V = -X \sin \alpha + Y \cos \alpha$$

Show that  $U$  and  $V$  will be uncorrelated if

$$\tan 2\alpha = \frac{2r\sigma_X\sigma_Y}{\sigma_X^2 - \sigma_Y^2}$$

where  $r$  = correlation coefficient of  $(X, Y)$ ,  $\sigma_X^2 = \text{var}(X)$  and  $\sigma_Y^2 = \text{var}(Y)$ .

- (b) A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies. 3+2

### GROUP-B

**Answer Question No. 5 and any two from the rest**

5. (a) Find the centre of gravity of a uniform arc of a circle. 2  
 (b) Show that a force and a couple cannot produce equilibrium. 2  
 (c) Write down the principle of virtual work for a single particle. 1
6. (a) Find the centre of gravity of the segment of a solid sphere of radius ' $a$ ', cut off by a plane situated at a distance ' $c$ ' ( $< a$ ) from its centre. 5  
 (b) Find the equation of the central axis of any given system of forces. 5
7. (a) A thin hemispherical bowl of radius ' $b$ ' and weight ' $W$ ' rests in equilibrium on the highest point of a fixed sphere of radius ' $a$ ' which is rough enough to prevent any sliding. Inside the bowl a small smooth sphere of weight  $W_1$  is placed. Show that the equilibrium is unstable unless  $W_1 < W \left( \frac{a-b}{2b} \right)$ . 5  
 (b) A rectangular hexagon is composed of six equal heavy rods freely jointed together, and two opposite angles are connected by a string which is horizontal, one rod being in contact with a horizontal plane; at the middle point of the opposite rod is placed a weight  $W_1$ ; if  $W$  be the weight of each rod, show that the tension of the string is  $\frac{3W + W_1}{\sqrt{3}}$ . 5
8. (a) Two forces act, one along the line  $y=0, z=0$  and the other along the line  $x=0, z=c$ . As the forces vary, show that the surface generated by the axis of their equivalent wrench is  $(x^2 + y^2)z = cy^2$ . 5  
 (b) Three forces  $P, Q, R$  act along the sides of a triangle formed by the lines  $x + y = 3, 2x + y = 1$  and  $x - y + 1 = 0$ . Find the equation of the line of action of the resultant. 5

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